COORDINATION, PRE-PLAY RESEARCH IN A MODEL OF BANK RUNS

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Abstract

Multiple equilibria in models of bank runs imply that coordination of play is a challenging task. In particular, for first-play type of situation, successful coordination appears a very strong assumption. We develop a simple representation where depositors tentatively select actions profiles that need not be consistent but can perform research in order to learn about others’ intentions. We show that changes in fundamentals influence the value of learning about others’ intentions in a way that can facilitate multiplier effects. This effect can be further amplified by the strategic complementarities in research activities. We identify cases in which the equilibrium is unique. En general, due to strategic complementarities, multiplicity cannot be ruled out. We explore the predictions associated to a form of procedurally rational behavior.

1. Introduction

Multiple equilibria in models of bank runs provide an important message for the interpretation of economic events. They indicate the existence of circumstances in which the mapping from actions to payoffs does not lead to an unambiguous prediction about actions selected by players. This intrinsic under-determination implies that agents playing the game face a tough decision problem in which reasoning about own and others’ incentives is not enough to assess the right action. In other words, this intrinsic under-determination implies that coordination of actions towards a consistent profile might not occur.

These comments are particularly relevant for situations in which the history of the game is nonexistent (first play) or non-stationary settings in which, after an important shock, past play is not a reliable guide of future decisions. For this relevant class of settings, the usual argument of coordination as a result of a dynamic process of convergence does not apply.

We present an analysis of a bank run game in which the initial profile of tentative play is not necessarily a consistent set of plans. Complementarily, we assume players can collect information about other players’ likely actions. More specifically, before play, they can receive a costly signal about the tentative play of others’. This signal allows for improved coordination. In this work, we study the strategic properties of this game and discuss prediction of play and outcomes.

We show that changes in fundamentals influence the incentives to learn about others’ intentions allowing for potential multiplier effects. This effect can be further amplified by the strategic complementarities in research activities. We identify regions in which the equilibrium is unique and, as a result, coordination of play in this meta-game might be more likely to occur. Nevertheless complementarities in research imply that, in general, multiplicity cannot be ruled out. With those cases in mind, we present and evaluate distribution of outcomes under behavioral assumptions associated with procedural rationality. Overall, we believe that, notwithstanding its exploratory nature, the type of analysis developed in this work can advance our understanding and refine our predictions of how actions are selected in contexts where strategic uncertainty implies consistent plans might not be selected.
The analysis of games with multiple equilibria has naturally led to debates on how the complex coordination problem is solved. One possible solution implies considering that there exist “sunspots” through which agents coordinate towards specific profiles of consistent plans. This explanation is not satisfactory since it amounts to ignoring or assuming away with the challenges posed by coordination. In addition, this explanation fails to provide a link between payoff structure and equilibrium play. Alternatively, it has been argued that equilibrium is selected according to evaluations of each equilibrium associated levels of risk and payoff. While considerations of average payoffs and risk are likely to influence behavior, these arguments on equilibrium selection need to be accompanied by a description of the process through which agents agree on the valuation of different equilibria.

In a different approach, coordination toward equilibrium has been explained as a result of a dynamic process in which agents learn about other’s likely play as they interact. These analyses typically assume a stable environment that allows for convergence to equilibrium. As indicated above, our description is focused on a different type of scenario. We focus on the case in which history is non-existent or not very informative. Nevertheless, we envision that there is a potential for the incorporation of our insights in dynamic non-stationary models.

Another strategy consists in representing this interaction as a game of incomplete information. In this way, the absence of common knowledge, can lead to uniqueness of equilibrium (see Postlewaite and Vives 1987, Rochet and Vives 2004, Goldstein et al. 2005). Players observe their signal and select best responses given the distribution of other players’ information and their contingent plans which in equilibrium are consistent with this best response. Under this scenario our opinions on the limits to coordination, while modified, still remain. The elimination of multiplicity might reduce the strategic uncertainty in one direction, but the introduction of asymmetric information poses new challenges for the emergence of coordination play.

The contributions focusing on communication are the most closely related to our contribution. These contributions typically consider situations in which two players have different preferences about the preferred equilibrium. In an enlarged game, messages are sent about intended play in order to coordinate play. Similarly, our formulation considers an enlarged game in which players learn about the intention of play others. But in our formulation we have many players, and learning takes the form of a noisy signal of rivals’ play that cannot be manipulated by a sender.

We believe that the insights presented in this analysis are also relevant in other economic interactions. In particular in circumstances where there exist multiple equilibria and previous play is not available or is not expected to be a reliable source of future play. Examples of economic phenomena where the presence of multiple equilibria imply that coordination might not be attained include currency attacks, arbitrage in financial markets, business cycles, bargaining and pro-social behavior.

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1 See Harsanyi and Selten (1988) for a comprehensive presentation.
2 See Fudenberg and Levine (1998)
3 Explanations based on the concept of a focal point can be viewed as incorporating both elements of sunspots and dynamic learning. Focal points permit predicting actions as a function of the shared opinions about the prominence of an option and independently of its payoffs. In this approach, coordination emerges as a result of reasoning about these shared opinions that are formed most likely through time by learning.
The next section presents a bank run game. Section 3 extends the basic model and analyzes the strategic properties of the resulting game. The following section focuses on the prediction of play and presents some numeric examples. Section 5 concludes.

2. Strategic Analyses in Bank Run Models

We consider a simple model of bank runs. There are $N$ depositors that simultaneously choose whether to renew or withdraw their funds. The amount of each deposit is equal to 1. If the amount of withdrawn funds is below $L < N$, then the depositors that renewed their deposits earn a payoff equal to $1 + d$ and those that withdrew their funds earn a return equal to 1. If the amount of withdrawn funds exceeds $L$, then the bank goes bankrupt. In this case the $N$ depositors that withdrew their funds receive a payoff equal to $z < 1$ and the payoffs for depositors that did not withdraw is 0. This is a simplified version that does not include impatient depositors (those that need to withdraw independently of the likelihood of bankruptcy). In addition we assume a very abrupt change in average payoffs implicitly associated with a large negative impact of bankruptcy on the value of the bank net assets.\(^5\) These simplifications are made in order to make the analysis more tractable, qualitatively similar results would hold for versions that are closer to the traditional model.

We introduce some notation by using $u_i(a)$ to represent the payoff function of player $i$ that is a function of $a$, the profile of actions. Where the $i$th component of $a$ equals $w$ if player $i$'s action is withdraw and $r$ if player $i$'s action is renew. Note that this is a symmetric game; alternatively, we could have defined a symmetric payoff function that depends on the action selected by $i$ and the action selected by the other players ($-i$).

Different methods can be used to analyze strategic interactions in this specific bank run game. Our main interest is placed in what could be referred as first play of the game. Alternatively, we could think of situations in which players face a large change in payoff functions that diminish the ability of previous play, and its associated learning process, to predict future action. In these situations, it is not obvious that successful coordination of play will result.

It is easy to verify that this game has two pure strategies Nash Equilibria. If all depositors withdraw, then withdrawing is a best response. In this case, the payoff associated with withdrawing is $z$, which is below one but higher than the null payoffs associated with renewing the deposit ($1 > z > 0$). If all players renew their deposit, then renewing is a best response. In this case the probability of bankruptcy is zero and there is a positive return for deposits ($1 + d > 1$). More generally, it can be observed that under bankruptcy withdraw ($w$) is the unique best response and renew ($r$) is the unique best response if the bank stays solvent. Thus, the two cases just described are the only two equilibria in pure strategies.

As already indicated in the introduction, we observe that this analysis does not provide any guideline about which of the two equilibrium action profiles is more likely. In addition, as long as there exist multiple equilibria, the result of this analysis does not change as the solvency of the system changes. In other words, no explanation is provided about how agents coordinate or select a specific

\(^5\) This payoff structure would be observed, for example, if the players in this game were junior debt holders. In this case, bankruptcy would lead to a violent negative adjustment in their payoffs, but conditioned on bankruptcy, their payoffs would not change significantly with the number of withdrawals.
equilibrium. The idea of sunspots serving as coordination device is equivalent to assuming the coordination problem is somehow solved.

One extension of this approach considers that players select equilibria based on a weighted evaluation of the payoff dominance and risk dominance properties of each option. While these properties are expected to influence play, it is difficult to visualize the process through a collective evaluation of the desirability of each characteristic leads to the emergence of an agreement that guarantees successful coordination of actions. To summarize, while the concept of multiple equilibria is an important construction, its very presence suggests that a satisfactory explanation of how play occurs requires further considerations and possibly different concepts. More directly related to our concerns, successful coordination toward consistent plans is a weak tool for the explanation or prediction of outcomes on the first play of a game.

An alternative approach for predicting outcomes is provided by assumptions consistent with procedural rationality, that is, the idea that players have goals but their behavior is generated by simple rules. Under procedural rationality, the use of simple rules captures the idea that agents do not have unlimited information processing resources in order to find the right action. One such option would involve assuming that players select the action that maximizes payoffs for a simulated play by rivals. In a simple formulation, the simulation would involve a single profile of rival’s play. In a further simplification, we could assume that rivals’ action profile is chosen randomly with equal probability over all possible profiles of rival’s actions.6 In our context, one possible interpretation of this choice rule is to think that in the absence of unambiguous information or inference about future rivals’ play, with no solid recommendation for a course of action, behavior responds to the relevant payoffs but with variation in the level of attention to different scenarios. This variation of attention generates variation in selected actions.

Clearly, the resulting behavior does not constitute a consistent profile of actions. On the other hand, play is responsive to payoff structure: more combinations of play that result in bankruptcy would lead, on average, to more withdrawals.

In contexts where the game presents significant intrinsic uncertainty (e.g. multiple consistent plans) this kind of assumption might generate a reasonable approximation to outcomes on a first play of the game.7 While for repeated interactions, we would expect that previous play would be incorporated in learning processes that generate behavior and could lead to consistent plans.8 But in real life, no interaction is an exact replica of previous situations. In particular, we have in mind interactions which have precedents but take place following a significant change that renders the history of play less relevant. It can be argued that extreme situations, such as interactions that have

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6 See Osborne et al. (1998) and Camerer et al. (2004) for related analyses. Osborne et al. (1998) considers situations in which players select strategies using independent samples of payoffs for each action. Camerer et al. (2004) consider situations in which players are divided into subsets with different levels of sophistication. Players in the first subset select each action with equal probability. Play in subsequent subsets is a best response to play in the preceding groups. Our prediction of play could be matched through a stochastic version of Camerer et al. (2004) with a large fraction of the players concentrated in the first three subsets. Additionally, our behavioral assumptions is equivalent to a stochastic version of the behavior of Camerer et al.’s second category (best response to rivals that choose randomly).

7 This interpretation with a focus on first period play is emphasized in Camerer et al. (2004).

8 See Camerer et al. (1999) for a flexible framework in which players learn from previous play. See also Fudenberg and Levine 1998.
the potential of leading to bank runs, constitute unusual events where play in recent history might not be as informative as in the case of normal unsurprising scenarios.

We now present the distribution of action profiles under the procedurally rational behavior described in the previous paragraphs. In this example we assume that the number of players is an integer number. Let, $\bar{N}$ be the smallest number of players that can generate bankruptcy by withdrawing their funds. That is: $\bar{N} = \min \{ n \in \mathbb{Z}: n \geq L \}$. Then, players choosing strategies according to the procedurally rational rule described above would withdraw their funds with probability equal to the fraction of rivals’ action profiles that would lead to bankruptcy:

$$\bar{p} = \frac{\sum_{k=\bar{N}}^{N} \frac{N!}{k!(N-k)!}}{2^N}$$

As a result, the probability that a profile of actions $\alpha$ occurs equals $Pr(\alpha) = \bar{p}^{w(\alpha)}(1 - \bar{p})^{(N-w(\alpha))}$ where $w(\alpha)$ is the number of players withdrawing in profile of actions $\alpha$, that is $w(\alpha) = \# \{ i \in N: a_i = \omega \}$. Similarly, the probability that a profile in which $k$ players withdraw occurs with probability $\frac{N!}{(k!(N-k)!)}\bar{p}^k (1 - \bar{p})^{(N-k)}$. Finally, the probability of default is given by:

$$Pr(w(\alpha) \geq L) = \sum_{k=\bar{N}}^{N} \frac{N!}{(k!(N-k)!)}\bar{p}^k (1 - \bar{p})^{(N-k)}.$$  

In this simple analysis, it is clear that a decrease in $L$ increases the probability of bankruptcy both directly and indirectly. The direct, or mechanical, impact is the increase in the set of action profiles that result in bankruptcy. The indirect impact is given by the change in the probability of each action profile. As $\bar{p}$ increases when $L$ falls, the action profiles with a high number of withdrawals are relatively more likely.

In figure 1 we provide numerical examples of the predictions associated with this approach. We first consider the case of $N = 8$ and $L = 5$. That is, there are eight players and bankruptcy occurs if 5 or more players choose withdraw. This example can be viewed as representing a situation in which large depositors (or more generally creditors) interact. Alternatively, this outcome could be generated by a case in which a large number of depositors are divided into a small number of groups and there is perfect correlation in the choices of players in each group.

The figures show two more cases, one in which bankruptcy occurs as long as 50% of the players withdraw and the last one in which the threshold is 75%. The figures show that the probability of bankruptcy changes greatly with the threshold levels because these changes affect the distribution of players’ choices. For example, for $L = 5$ the probability of choosing withdraw is 0.227 while for $L = 6$ the probability of choosing withdraw is 0.062. As a result, the probability of bankruptcy is approximately .018 in the first case while it is almost null in the second case.
Figure 1: Distribution of action under procedural rationality

Probability Mass Function for Number of Withdrawals (N=8, L=5)

- P(Bankruptcy) ≈ 0.018

Probability Mass Function for Number of Withdrawals (N=8, L=4)

- P(Bankruptcy) ≈ 0.63

Probability Mass Function for Number of Withdrawals (N=8, L=6)

- P(Bankruptcy) ≈ 0
Through this example we illustrate how simple, but reasonable assumptions on players’ behavior can lead to interesting predictions on possible outcomes. On the other hand, it could be argued that players might be interested in developing more refined processes to generate behavior. Motivated by this idea, we believe interesting insights can be gained by complementing the simple approach by allowing agents to take actions associated with their interest in coordinating play. Naturally, this is one selection among several other directions in which the players’ selection of actions could be refined. For example, other options include trying to persuade other players to follow an action or developing more refined inferences given rivals are acting based on simple rules.

So far, we have considered two strategies for predicting play in the specific bank run game under analysis. First, using equilibrium analysis we identified consistent plans. The second strategy involves assuming behavior consistent with procedural rationality. We observed that, for first period play type of situation, the first strategy for prediction is inadequate. On the other hand, the second strategy is one plausible account which incorporates both response to incentives and the challenges of coordination. Our analysis in the rest of the paper will consider a halfway scenario in which players do not achieve flawless coordination, but can perform activities in order to make progress in their ability to coordinate. This step will provide interesting insights on the determinants of the selection of action profiles.

3. The Extended Game: Pre-play research

In what follows, we develop an extension of the game. It can be interpreted as complementing the analysis under procedural rationality by allowing players to carry out activities that increase their ability to coordinate actions. In this game, players have a tentative profile of actions that need not be consistent but they are able to perform costly research activities that facilitate coordination. After presenting the game, we will describe its strategic properties. We will be able to verify that the value of learning about others’ tentative play can change significantly as a function of fundamentals (payoffs) and the research actions of other players. Importantly, these results are not based on specific equilibrium concepts, they simply indicate which is the welfare impact of such activity.

We extend the bank run game by introducing a pre-play stage in which agents can perform research activities in order to learn information about the tentative play of other players. The initial profile of tentative play \( a^0 \) is distributed randomly with probability distribution function \( F(a^0) \). At this point of the analysis we do not make additional assumptions about a specific probability distribution.

Research activities are developed simultaneously. In this pre-play stage, each player sets the level for an activity that increases her information about rival’s tentative play. Let \( b_i \in \{0,1\} \) represent the level of this research activity. In order to distinguish this activity from the action in the second stage, we will use the expression “research action” to refer to the choice of \( b_i \). If \( b_i = 1 \), player \( i \) will receive a message \( s_i \in \{w,r\} \), a noisy signal about the tentative action of rivals \( a^0_i \). The signal is \( w \) with probability equal to the fraction of rivals tentatively selecting \( w \) and, consequently, the signal is \( r \) with probability equal to the fraction of rivals tentatively selecting \( r \). Additionally, we assume that the signals received by players with different tentative play are independent. We assume: no signal is received if there is no research. Research activity is potentially costly and is associated to a negative impact on payoffs equal to \( c \geq 0 \).
The performance of costly research activities is carried out with the intention of allowing this information to affect behavior. In this binary setting, information can only be used by adopting the action indicated by the message. As a result, we assume that if an agent receives a signal, then the action indicated in the message is adopted. This can be interpreted as a simple rule of thumb; given the information content of the signal, this rule results in the correct response in most cases.

As a result, this modified version of the model is a game with asymmetric information where each agent has a type determined by its tentative action. Interestingly, differentiating our contribution from most analyses with asymmetric information, the asymmetry of information is not about payoff functions but about tentatively selected actions. An action is given by a level of research activity, \( b_i \in \{0,1\} \), and a strategy determines a selected action for each possible tentative play: \( B_i : \{r,w\} \rightarrow \{0,1\} \). The payoff function of this game is given by: \( U_i(a^1, b_i) = u_i(a^1) - cb_i \) where \( u_i(,) \) is the payoff function of the original game and \( a^1 \) is a function of the tentative play \( a^0 \) and the random outcome of research activity represented by profile \( B \). This last relationship is used in the computation of the expected payoff given by \( EU(a^0, B) = E(u(a^1) - cb_i | a^0_i, B) \).

We will evaluate the strategic properties of this game in ex-ante terms, that is, for a given probability distribution of profiles of tentative play. In terms of strategic properties, our discussion will gyrate around the welfare impact of research, strategic substitutability and strategic complementarity.\(^9\)

With respect to spillovers, we find that the welfare impact of research on others depends on the tentatively selected action of the player performing this activity. The spillover is negative in the case of players that have tentatively chosen to renew the deposit. This is because number of players that withdraw their deposits can only increase as players with tentative play \( r \) perform research activities that can change their tentatively selected action. This results in negative externalities as, for a fixed distribution of other players’ actions, the probability of bankruptcy increases. On the other hand, through a similar reasoning, we can verify that research by agents that have tentatively selected \( w \), leads to positive externalities.

In order to compute the value of researching others’ tentative play, we will compute the expected gains from getting a signal as a function of the tentative play \((w \text{ or } r)\). This calculation assumes an underlying distribution for profiles of tentative play \( (F(a^0)) \) and a profile of research activities of rivals \( (b_{-i}) \). We the rest of the paper, we assume the set of depositor is a continuum of size \( N \). As a result, for a given distribution, the number of players selecting each action will be unknown only if there is correlation in their choice of actions.

If a player that is tentatively choosing \( w \) receives a signal \( r \) and adopts this action, then she would gain \( d \) from doing research if the number of rivals choosing \( w \) is less than \( N \). On the other hand, if the number of players choosing \( w \) is larger or equal to \( N \), then she will suffer a loss of \( z \). The probability of receiving a signal \( r \) is a function of the profile of tentative play of rivals \( a^0_i \). Let \( w(a^0_{-i}) \) indicate the number of players selecting \( w \) in tentative profile of actions \( a^0_{-i} \). Then the probability of receiving signal \( r \) is given by: \( \frac{N-w(a^0_{-i})}{N} \). Let \( F_{-i}(w(a^0_{-i})) \) be the distribution of rivals’ tentative play. Finally, the profile of actions selected by rivals is a random variable with a distribution that depends on the profile of tentative play and the research activities of rivals: \( Pr(a^1_{-i} | a^0, b_{-i}) \).

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\(^9\) For a seminal paper see Bulow, Geanakoplos and Klemperer (1985).
Given these definitions, the expected welfare effect following signal \( r \) given an original tentative choice of \( w \) is given by:

\[
\Delta EU(w) = \sum_{w(a_{-i})=0}^{N-1} F_{-i}(w(a_{-i}^0)) \left[ \frac{N - w(a_{-i}^0)}{N} \left( \Pr(w(a_{-i}^1) < \bar{N}|a^0, B_{-i})d - \Pr(w(a_{-i}^1) \geq \bar{N}|a^0, B_{-i})z \right) \right]
\]

The first term inside brackets captures the gains associated with the contractually established positive return of the deposit while the second term reflects the losses associated with bankruptcy. Similarly, the net gains from receiving a signal \( w \) when the original tentative play is \( r \) is given by:

\[
\Delta EU(r) = \sum_{w(a_{-i})=0}^{N-1} F_{-i}(w(a_{-i}^0)) \left[ \frac{w(a_{-i}^0)}{N} \left( \Pr(w(a_{-i}^1) \geq \bar{N}|a^0, B_{-i})z - \Pr(w(a_{-i}^1) < \bar{N}|a^0, B_{-i})d \right) \right]
\]

Observe that the research activities of rivals have an impact only through the distribution of \( w(a_{-i}^1) \). That is, changes in \( B_{-i} \) do not have impact on the messages received. We can now show some results regarding the strategic properties of this game.

Consider the hypothetical case in which starting from an arbitrary plan for research activity \( B_{-i} \) we consider a new profile \( B'_{-i} \) where the research level associated to tentative play \( w \) is, for each rival, equal or higher than those observed in the original profile. That is, \( b'_{-i}(w) \geq b_{-i}(w) \) and \( b'_{-i}(r) = b_{-i}(r) \) for all \( j \neq i \). Then, we can establish that, for any given profile of tentative play \( a_{-i}^0 \), the new conditional distribution for \( w(a_{-i}^1) \) is stochastically dominated by the older distribution in the first order sense. That is, the conditional distributions satisfy:

\[
\Pr(w(a_{-i}^1)|a^0, B'_{-i}) \geq \Pr(w(a_{-i}^1)|a^0, B_{-i}) \text{ for all } a^0 .
\]

The result follows from observing that as more players with a tentative play of \( w \) receive a signal, then36 for any initial profile of tentative play \( a^0 \), the new distribution of the quantity of those that keep their intention to play \( w \) is dominated by the older distribution. Note that, for a given \( a^0 , w(a_{-i}^1) \) equals the sum of two independent random variables: the number of those that had tentatively selected \( w \) and did not receive signal \( r \) and those that had tentatively selected \( r \) and received a signal \( w \). Since the distribution for the second component remains unchanged and the new distribution of the first component is stochastically dominated in the first sense by the older one, we find that the new random variable that results from the sum is stochastically dominated by the older one in the first sense.

After having established that \( \Pr(w(a_{-i}^1)|a^0, B'_{-i}) \geq \Pr(w(a_{-i}^1)|a^0, B_{-i}) \) for all \( a^0 \), it is simple to inspect equation (1) and verify that the expected gains increase as a result of the change in the research activity profiles. This is because as a result of the change in these probabilities, for any initial profile \( a^0 \), the probability of a gain of \( r \) is now higher and the probability of a loss of \( z \) is now lower. On the other hand, the resulting new distribution for \( w(a_{-i}^1) \) has the opposite effect on the net gains from doing research by players with initial tentative play \( r \). For these players, research by players with \( w \) as their tentative play increase the likelihood of losing \( r \) and decreases the likelihood of gaining \( z \) for any initial profile \( a^0 \).
Similarly, it can be shown that increases in research activity by players with a tentative play \( r \) would increase the returns to research by players of tentative play \( r \) and decrease the net gains from research by players selecting \( w \). This is because for research activities profiles \( b'(w) = b_i(w) \) and \( b'_j(r) \geq b_j(r) \) for all \( j \neq i \), the associated distributions for \( w(a^{1}_{-i}) \) are such that

\[
\Pr (w(a^{1}_{-i})|a^0, B'_{-i}) \geq \Pr (w(a^{1}_{-i})|a^0, B'_{-i}) \text{ for all } a^0. \]

We can summarize our observations in the following proposition:

**Proposition:**

Research activity is strategic complement for players with the same tentative play and strategic substitute for players with the different tentative play.

These results imply that properties of the extended game are such that, in general, multiple equilibria cannot be ruled out. That is, for certain parameter values, the observations we made on the challenges posed by coordination can potentially reemerge for the case of the extended game. Players would find it hard to select research activity plans such that a profile of consistent choices results.

We can also develop some simple comparative statics of the value of research. In particular, consider the case of a change in the level of withdrawals necessary to lead to bankruptcy (\( \bar{N} \)). In this case, we can observe that, for any profile of research plans by rivals, a fall in \( \bar{N} \) would increase the value of doing research conditioned on a tentative action \( r \). This is because a fall in \( \bar{N} \) would result in lower \( \Pr (w(a^{1}_{-i}) < \bar{N}|a^0, b_{-i}) \) and, consequently, a higher probability of gaining \( z \) and lower probability of losing \( d \). It is easy to check that the opposite relationship holds for the value of doing research by players that have tentatively selected \( w \). In addition, a fall in \( \bar{N} \) would affect the net gains from doing research in the same direction. We summarize these observations in the following proposition:

**Proposition**

A fall in the minimum number of withdrawals that generates bankruptcy (\( \bar{N} \)) or fall in the payoffs associated to withdrawing followed by bankruptcy (\( z \)) result in higher value of doing research by players that have tentatively selected \( r \) and lower value of doing research by players that have tentatively selected \( w \).

This very simple result provides a simple illustration of a multiplier effect of changes in fundamentals. Changes in the game payoffs lead to changes in the value of learning about others’ likely play. The value depends on the tentatively chosen action. Worse fundamentals lead to higher value for doing research by players that are tentatively optimistic and lower value for doing research by players that are tentatively pessimistic. If this effect is considered jointly with the previous results on the strategic complementarities in actions, the multiplier effect gains more strength as more research leads to higher value for doing research.
Throughout this simple analysis, we have made observations on changes in the value of research activities. We have not considered the prediction of play and we have kept the probability distribution for $d_0$ constant. In the next section, we will consider alternatives for predicting play. We will present numerical examples illustrating different outcomes that can emerge in the extended game.

4. Prediction of Outcomes and Numerical examples

So far, we have focused on describing the properties of the extended game for a fixed distribution of tentative play profiles. More specifically, our analysis of the extended game has shown how the value of learning about rivals’ play depends on fundamentals (payoff functions), others’ research activity and own tentative play. This analysis was developed assuming a fixed distribution for the profile of tentative actions. These results are significant by themselves, but we are also interested in analyses that explain how play is expected to take place in this game.

Before advancing, we would like to make an observation about the interpretation of the extended game. Players have a tentative intention of play and make a decision on doing research about others’ play. We can think of these two elements as the result of two separate but related processes that generate actions based on assessments of the value of each option. As we illustrated in section 2, tentative play can result from a simple simulation of the game payoffs. Given the intention of play, the evaluation of the decision to do research is supposed to be the result of an independent process. For example, it could also involve a simple simulation of play. The separate nature of these two elements allows for instances in which a large fraction of players with a given tentative action carry out research that result in ex-ante welfare gains.

In order to study the selection of actions and outcomes in the extended game we need to specify a process for the determination of tentative play and a rule for determining choice of research activities. With respect to tentative play, $a_0$, we will consider the procedurally rational process described in section 2 for the original bank run game. That is, we will assume that the probability that player $i$ selects action $w$ is equal to the fraction of possible profiles of rivals’ actions for which $w$ is a best response.

With respect to choices of research activities, as in the original game, we identify two types of lines of attack. The first class consists in finding the set of consistent plans. We believe this exercise is informative; for example, the existence of unique or multiple equilibria is one factor influencing the ability of players to coordinate their actions toward a consistent play. Nevertheless, given our emphasis on first-play situations, we find that this is not an adequate tool for predicting play. The arguments presented in the discussion of the prediction of play in the original game are also valid for the extended game. In addition, the consideration of the extended game is motivated by an interest in considering the limits in players’ ability to coordinate actions in the original game, then it would be contradictory to assume consistent plans in this stage.

With these considerations in mind, we will evaluate outcomes of this game using a second type of approach associated to procedural rationality. We will assume agents use simple rules to assess the value of different options. More specifically, we assume players select actions based on the result of
a simple simulation of play. The simulation involves evaluating the payoffs associated to a randomly chosen combination of rivals’ research decisions.\(^\text{10}\)

In our numeric examples we will assume that there are \(K\) types of players with a continuum of \(K/N\) players in each class. Players in the same class have the same tentatively selected action and, in the case they perform research, all of them receive the same signal. This a convenient formulation in which players tentative actions are correlated and in this way there is aggregate uncertainty despite the large number of players. We could interpret this assumption as a reduced form of the model with social interactions, in which agents that interact frequently share mental models that generate variation in tentative play and signals about others’ play.

We will present two examples. In the first one we show that in spite of complementarities, there are some circumstances in which the extended game has a unique equilibrium. The second example will show how changes in fundamentals are associated with changes in the distribution of outcomes.

Example 1: Unique equilibrium

Consider parameter values: \(N = K = 6, L = 3, z = .5, d = .05\) and \(c = 0\). Through a numerical computations we can show the uniqueness of equilibrium for the extended game in which players decide on research activity and tentative play is a determined randomly by the simple simulation of play algorithm described above. Below we provide a table that shows the value of doing research as a function of the tentative play and the decisions of other players:

\[\begin{array}{c|cc}
 b(w) & b(r) & 0 & 1 \\
 (\Delta EU(w), \Delta EU(r)) & (-0.14,0.14) & (-0.18,0.21) \\
 0 & (-0.07,0.11) & (-0.09,0.15) \\
 1 & & \\
\end{array}\]

These calculations of the value of performing research activity show that for this specific example, performing research by players that have tentatively selected \(w\) is a dominated strategy. The value is negative in the case in which players with tentative action \(r\) do not perform research activities and players with tentative action \(w\) do research. Due to the strategic complementarities and strategic substitutabilities we demonstrated in the previous section, this negative value means that for any other combination of other players research levels the value of doing research is negative.

Additionally, for players with tentative action \(r\), the value of doing research is positive for all combinations in the table. In particular, the value is positive when all players with tentative action \(w\) select doing research and all other players with tentative action \(r\) do not select to do research. Due

\(^{10}\) As indicated in section 2, we could also consider that a fraction of players are more sophisticated than the players that select a best response to a randomly determined combination of rivals’ play. Qualitatively the results would not change, but, for example, this modification could result in stronger multiplier effects.
to the strategic complementarities and substitutabilities, this observation implies that for players
with tentative action \( r \) doing research dominates not doing research.

Through the elimination of dominated strategies, we verified that in this case the game has a unique
equilibrium in which players with tentative action \( r \) choose doing research and players with
tentative action \( w \) select not doing research.

We observe that in this case in which there is one strategy that is dominated, then the equilibrium
play will coincide with the outcome associated to the selection of actions through the simple
simulation of play we proposed. In our simple example, equilibrium outcomes would be observed
even if players employ a simple, incomplete rule to determine their action. We have shown this
results in the case in which fundamentals are relatively weak (high \( L \)) similar results but with the
opposite selecting of actions would hold in the case in with fundamentals are strong (\( N \approx L \)).

Example 2: Changes in payoffs and multiplier effect

Consider initially the case in which \( N = K = 6, L = 5, z = 0.5, d = 0.05 \) and \( c = 0 \). In this case,
assuming that tentative play and the decision of doing research is based on the simple simulation of
play we find that the distribution of outcomes is such that the probability of bankruptcy is
approximately 0. The first panel in figure 3 shows the distribution of the number of withdrawals in
the tentative profile and the profile that results after research activities are carried out. The
distribution of tentative profiles is tilted toward low levels of research and the probability of
bankruptcy is already approximately null under this distribution. The evaluations of research
decisions based on the simple simulations generate high activity by players tentatively selecting \( w \)
and lows levels by players tentatively selecting \( r \). As a result, the new distribution of action profiles
is more tilted toward low levels of withdrawals. With a probability higher than .95 the observed
outcomes will have all players selecting \( r \).

In the second panel we consider a change in \( L \), we set \( L' = 4 \). In this case there is an increment in
the number of tentative withdrawals, as a result the probability of bankruptcy based on tentative
play is approximately 0.013. Research activity, as shown in the second panel of figure 3, has an
important impact on the distribution of action profiles, but it does not have an impact on the
probability of default. This is because on average, players having tentatively selected \( w \) have a
higher propensity to select to do research than players with tentative action \( r \), 60% versus 50%. As
a result, there is a higher degree of coordination, the weight of profiles in the extremes increases.

Finally, we consider \( L = 3 \). In this case there a large increase in the probability of bankruptcy as a
combined result of the direct effect through the change in the threshold, the higher propensity of
players to tentatively select \( w \) and the research activity by players that have tentatively selected \( r \).
The effect of research on the change in the distribution of selected action profiles is visible in panel 3
of figure 3.
Figure 3: Probability Mass Function for changes in $L$ (N=6, z=.5, r=.05)
The multiplier effect can be observed by comparing the distribution of outcomes for the middle panel in figure 3 with the distributions observed in the lower panel. In this case, research activity moves the distribution further away from what is observed in the middle panel and toward the right region of the chart. As a result, there is a large change in the distribution of outcomes after research observed in these two panels.

In these examples we assumed the cost of doing research are null. Naturally are results can be greatly affected by manipulating this parameter. Just to mention one example, if the value of research is positive, then multiple equilibria result as long as the value of this cost level is set in an appropriate range. This is a consequence of the complementarities that result from the properties of the value of doing research.

6. Concluding Remarks

We have analyzed a simple game of bank runs. We have argued that, for first-play type of situations interesting insights can be gained by considering a scenario in which players determine tentative play based on simple rules, resulting in potentially non-consistent tentative plans that can be revised by performing activities that permit learning about others’ tentative play. We gained insights on variations in the value of this information. We also showed, through numerical examples, the type of outcomes that would be expected if players follow simple rules to determine their tentative play and their research activities.

This analysis naturally demands empirical exercises to evaluate the fitness of the predictions. Experiments seem to be the most natural environment where coordination and research efforts by participants playing the game on a first occasion can be evaluated.

Another direction in which our analysis can be further developed has to do with exploring richer dynamics. Throughout this work we have emphasized that first-play type of situations was the environment that we have in mind. But the insights we have gained can be further explored together with models of adaptive learning. Non-stationary conditions, in particular contexts of repeated interaction where payoffs can change abruptly is definitely other setting where perspectives similar to the ones we developed in this work could be applied.

Finally, two areas in which this type of analysis can be further developed are the consideration of public signals and diversity in the level of sophistication of procedurally rational players. Public signals can play an important role in facilitating coordination. Regarding levels of sophistication, we have assumed that all players use a similar type of simple rule to select actions. More sophisticated rules would involve further inferences regarding the value of an action given the rules used by other players. In some cases, additional insights can be gained by assuming that players are characterized by, or can select, rules with different levels of complexity to select appropriate actions.

7. Bibliography


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