Abstract

This is a first step towards the optimal unemployment insurance design in general equilibrium model moral hazard. It considers a self-financed policy in the context of a simplified version of Lucas-Prescott (1974) with unobservable search effort. In this context, existence of an optimal unemployment benefit is shown. By definition, this unemployment insurance generates a steady state equilibrium with strictly higher social utility relative to laissez-faire. As a consequence of the trade-off between insurance and incentives to search, the equilibrium with the unemployment insurance has a strictly higher unemployment rate, relative to the laissez-faire equilibrium.
Moral Hazard and Optimal Unemployment Insurance: A General Equilibrium Analysis*

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1 Introduction

A main component of actual welfare policies in developed economies is the unemployment insurance program. Shortly, unemployment insurance is a transfer of money from the government to the unemployed agents while they search for reemployment. To gain access to these transfers, the unemployed agents have to possess certain characteristics, in particular, they have to demonstrate that, although they are currently jobless, they are actively looking for a job. When a worker becomes jobless, his labor income disappears, and assuming that the unemployed has no other alternative source of income, the same would happen to his feasible consumption. This is one of the essential reasons why the unemployment experience is a very traumatic one.

Through the unemployment insurance policy, the government has the objective of helping unemployed workers reduce the cost of being unemployed (in terms of reduced consumption), providing an alternative income source while they search for a new job. Literally, the government is providing an insurance, reducing the uncertainty in consumption that unemployment generates. The unemployment subsidy, however, creates a moral hazard problem. Just because the subsidy contingent on being unemployed reduced the cost of being unemployed in terms of reduced consumption, the unemployed agents have less incentives to search for a new job. This should increase the number of unemployed agents in equilibrium. When the cost of being unemployed is reduced, the unemployed agents will show a higher tendency to remain unemployed while they receive the insurance. Although one of the basic conditions to gain access to unemployed benefit is to demonstrate that the agent in actively searching for a new job, a typical moral hazard behaviour is evidenced when the unemployed, because of being insured, reduces his effort in the job searching activity. This behaviour that cannot be observed (or alternatively, cannot be enforced) by the government. As the probability of receiving a job offer depends positively on the job searching effort, if the unemployed reduces his effort, his unemployment spell will be longer.

The objective of this paper is to design an optimal unemployment scheme in a general equilibrium framework, solving the moral hazard problem associated to these contracts. In this paper, I study the design of an optimal unemployment insurance in the context of a general equilibrium model. This scheme consists of positive or negative transfers between a principal and the rest of the agents (workers) of the economy, contingent to the employment state of the agent, so as to solve optimally the trade-off between income insurance and the incentives problem.

1.1 Related papers

In their work on unemployment benefit, Shavell and Weiss (1979) show that to solve optimally the moral hazard problem associated to the unemployment insurance contracts the transfers to unemployed agents should decrease monotonically with the length of the unemployment spell. Hopenhayn and Nicolini (1997) complete this idea with a partial equilibrium model showing that, in order to correct the moral hazard problem, the optimal unemployment insurance contract should include a scheme of unemployment benefits that decrease monotonically through time, complemented by an employment tax after the agent has
found a new job. Hopenhayn and Nicolini, in a partial equilibrium framework, show under standard conditions that the employment tax rate should increase with the length of the preceding unemployment spell. The unemployment insurance consists of transfers between one principal, the government, and one unemployed agent, without considering what is the incidence of the unemployment insurance in the rest of the agents in the economy (other unemployed agents, employed agents and the firms). The design of the optimal unemployment insurance contract is developed in a partial equilibrium framework and, as a consequence, for practical purposes, we would have to repeat this individual contract, without taking into account the effects over the market equilibrium. Although the trade-off between income insurance and moral hazard behaviour is optimally solved, the incidence of the policy in a general equilibrium is (by construction) ignored.

Our model is a first attempt of an extension of these ideas to a general equilibrium model. This is basically a simplified version of the well-known Lucas and Prescott’s island model (1974). This assumes an economy composed by a spatially differentiated markets, which coexist simultaneously. The model supposes only one productive factor, labor, remunerated competitively by its marginal productivity (the wage). The spatial separation between islands allows the labor to be remunerated differently at the same time without abandoning the perfect competition assumption. In the model to be developed, there are only two types of islands, one corresponding to employment and the other to unemployment. Alvarez and Veracierto (1999) use the same model to evaluate quantitatively the effects of different types of policies in the labour market, among which they include unemployment benefit. Moreover, they suppose that the unemployed agents that decide to look for a job cannot choose the amount of their job searching effort. Hence they receive a job offer in a completely random way. Their laboral search is “undirected”, in the sense that the unemployed agents cannot influence in any way in the outcome of their job search.

Atkenson and Lucas (1995) also design an optimal unemployment insurance in an economy with incomplete information. In their model, workers have private information about their laboral history, which is unobserved by the government. Their optimal unemployment insurance minimizes the cost of provision of the unemployment insurance scheme which incentives every agent in the economy to self-reveal their true labor history. The problem in our paper is completely different from the one treated in Atkenson and Lucas, since here the focus is on the trade-off between labor search effort and consumption smoothing of workers.

1.2 Summary of contributions

My contribution consists of incorporating the individual decision of an unemployed agent over how much effort to devote to job searching activity in a specific general equilibrium model, the island model. If an unemployed agent devotes a higher searching effort to look for a new job, his job searching costs will be higher, but it will be more likely that he finds a job in the next period. On the other hand, if the unemployed agent reduces his job searching effort, he will reduce his job searching costs, but he will also reduce the probability of receiving a job offer next period. In this case, the job searching activity is directioned by the unemployed agent’s effort.
I consider first a laissez-faire equilibrium, where unemployed (and employed) agents do not receive or pay any kind of transfer to the policymaker. As a consequence, the unemployed agents will not be able to consume until they find a job. I concentrate in the steady state solution to this economy. Second, the features of an (restricted) optimal unemployment insurance system are defined. This contract has to obey the following restrictions:

- The optimal unemployment insurance consists in a transfer to unemployed agents and a tax to employed agents for one period.
- The insurance endogenizes the trade-off between insurance of income and incentives to job search while unemployed.
- This policy is self-finance, which means that the tax charged to employed agents will match exactly the benefit paid to unemployed agents.

Under very general conditions, I show that this optimal unemployment insurance exists, and under certain restrictive conditions may be potentially provided by private institutions. It is shown that, as a natural consequence of the trade-off between incentives to job search and insurance, both the unemployment rate and the unemployment duration increase in the presence of the optimal unemployment insurance.

The paper proceeds as follows. Section 2 develops the basic model used throughout the paper. In section 3, the steady state laissez-faire equilibrium of this model is characterized. Section 4 is the core section of the thesis, where the self-financed unemployment insurance contract is described and its impact on the steady state equilibrium is studied. At the end of section 4, I demonstrate the existence of the self-financed optimal unemployment insurance. Section 5 closes the paper with the final remarks and possible extensions.

## 2 The Economy

The economy is populated by a measure one of ex-ante identical agents with preferences given by:

\[
U_i(\{c_{i,t}, e_{i,t}\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t [u(c_{i,t}) - C(e_{i,t})]
\]

where \(c_{i,t}\) is agent’s consumption of the non-storable market good and \(e_{i,t}\) is the amount of effort devoted to job searching while unemployed. It is assumed that there is only one marketable good available in this economy, and the consumption of any agent is determined by the addition of the wage measured in market goods \(w_{i,t}\) and the transfer of market goods from the policymaker to agent \((z_{i,t})\).

\[
c_{i,t} = w_{i,t} + z_{i,t}
\]

In the laissez-faire equilibrium, the transfer of market goods from the policymaker to any agent will be zero by definition.

The function \(u(c)\) is the period utility of consumption of market goods. \(u(c)\) is assumed to be continuous and differentiable of order at least two, strictly
increasing and strictly concave. Besides, when the consumption of market goods
tends to zero, the period utility is finite, and the marginal period utility is
infinite. The function \( C(e_{i,t}) \) represents agent’s instant cost of job searching
with \( e_{i,t} \) effort, measured in market good. \( C(e_{i,t}) \) is assumed to be continuous
and differentiable of order at least two, and also increasing, strictly convex for
every agent \( i \). We also assume zero marginal cost when the job searching effort
is zero. Let \( \beta \in (0, 1) \) be the discount factor.

The market good is a non-storable good produced by a continuum of islands.
Each island has a production technology given by:

\[
y_t = F(n_t) = \omega n_t^\sigma
\]

where \( y_t \) is the output produced, \( n_t \) is the labor input, \( \omega \) is a positive pro-
ductivity parameter and \( \sigma \) is a parameter corresponding to the labor elasticity,
which is assumed to take a value in the interval \( \sigma \in (0, 1] \).

We assume that there is perfect competition between islands and, therefore,
the profit maximizing policy for firms is that each agent employed is paid a wage
\( w_{i,t} \) equal to his marginal productivity:

\[
w_{i,t} = w(n_t) = \frac{\partial F(n_t)}{\partial n_t} = \omega \sigma n_t^{\sigma - 1}
\]

To simplify, we assume constants returns to scale in the firms’ production
technology, \( \sigma = 1 \). With this assumption it is clear that \( F(n_t) = \omega n_t \). Hence
\( w_{i,t} = \frac{\partial F(n_t)}{\partial n_t} = \omega \) for every \( i \) and every \( t \).

At the end of each period, an agent employed can be fired from his job (ex-
pulsed from the island) with a probability \( \phi \). In this case he will be unemployed
in the following period. He can keep his current job with probability \( (1 - \phi) \).
In this case he will be employed in the following period in any of the islands
with productivity \( \omega \). We are assuming that the employed agents, who have the
option of abandoning their job, prefer not to do so. This will be a result in both
the equilibrium with and without unemployment benefit.

We can write the Bellman equation for the employed agent’s problem at the
beginning of period \( t \) as follows:

\[
V_E = u(z_{E,t} + \omega) + \beta [(1 - \phi)V_E + \phi V_U]
\]

if the employed agent receives a transfer of \( z_{E,t} \) market goods from the policy-
maker at time \( t \).

The unemployed agent can decide the search effort level, and, hence his job
search is directed. A higher search effort is rewarded with a higher probability
of receiving a job offer in the following period but, at the same time, implies
a higher search cost. Non-employed agents who decide to actively search for
a job will be the unemployed. We take non-employed agents as synonymous of
unemployed, assuming that in equilibrium, the non-employed agents will actively
search for a job. This will also be a result in both equilibriums with and without
unemployment benefit. We will assume that the agent has no other income source
than the wage paid by the firms.

In this model, the non-employed worker’s problem is equivalent to the fol-
lowing Bellman equation:
\[ V_U = \max_{e_{i,t} \geq 0} \{u(z_{U,t}) - C(e_{i,t}) + \beta[P(e_{i,t})V_E + (1 - P(e_{i,t}))V_U]\} \]

where \( V_U \) is the maximized discounted utility of a non-employed agent, which receives a transfer of \( z_{U,t} \) market goods from the policymaker at time \( t \). We define the variable \( e_{i,t} \), a non-negative real number, as the effort involved in the job searching activity, \( C(e_{i,t}) \) as the cost of searching a job with an effort of \( e_{i,t} \) and \( P(e_{i,t}) \) as the probability of finding a job in the following period if the agent has devoted an amount \( e_{i,t} \) of effort on looking for a new job in the current period. According to the preceding definitions, for every value of \( e_{i,t} \) and for every agent, the cost is nonnegative and the probability of reemployment is a bounded number between zero and one (\( P(e) \in [0,1] \)).

Assume also that if the agent makes no effort he will have no searching cost (\( C(0) = 0 \)), but then he will receive for sure no job offer (\( P(0) = 0 \)). On the other hand, if an agent devotes infinite job searching effort the cost of job searching is also infinite (\( \lim_{e \to +\infty} C(e) = +\infty \)), but then he will receive for sure a job offer (\( \lim_{e \to +\infty} P(e) = 1 \)). We assume that the cost of job searching activity and the probability of finding a job in an island with a high productivity job are both differentiable and strictly increasing functions of the job search effort. Let \( e_{i,t} \) be the best response of an unemployed agent, that is, the optimal search effort level for the unemployed agent:

\[ e_{i,t} = \arg \max_{e_{i,t} \geq 0} \{u(z_{U,t}) - C(e_{i,t}) + \beta[P(e_{i,t})V_E + (1 - P(e_{i,t}))V_U]\} \]

### 2.1 The Distribution of Agents

In this economy there are two possible individual states: employment or unemployment. There is no aggregate uncertainty. Then, the distribution of agents in the economy is determined by the state of the distribution the period before and the probabilities of transition between states from the period before into the current period. The transition probabilities will be given by the probability of firing if employed and the probability of receiving a job offer if unemployed.

Because every agent in the economy can either be employed (\( E_t \)) or either cannot (\( U_t \)), the following equation must hold:

\[ U_t + E_t = 1, \quad \forall \ t \]

Employed agents have a probability \( (1 - \phi) \) of remaining in their job, whereas unemployed agents have probability \( P(e) \) of receiving a job offer. We can deduce the following transition equation for employment:

\[ E_{t+1} = P(e)U_t + (1 - \phi)E_t, \quad \forall \ t \]

Employed agents have a probability \( \phi \) of getting fired, whereas unemployed agents have probability \( (1 - P(e)) \) of remaining jobless. We can deduce the following transition equation for unemployment:

\[ U_{t+1} = (1 - P(e))U_t + \phi E_t, \quad \forall \ t \]

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This assumes that every agent wants to keep his job or is looking for one, the whole continuum of measure one of agents is economically active. As mentioned above, this is in fact a result from the model. Therefore $U_t$ can be naturally interpreted as the unemployment in period $t$ and $E_t$ as the employment in period $t$.

2.2 The Steady State Distribution of Agents

We are interested in the steady state version of this economy’s equilibrium. Therefore, we can rewrite the steady state version of the equations that determine the distribution of agents:

\[
\begin{align*}
1 &= E + U \\
E &= P(e)U + (1 - \phi)E \\
U &= (1 - P(e))U + \phi E
\end{align*}
\]

where $P(e)$, in this case, is the probability of getting employed when the optimal job searching effort is being made, given the current unemployment compensation. From here it easy to get the steady state equilibrium unemployment:

\[
U = \frac{\phi}{\phi + P(e)}
\]

This implies a couple of very intuitive results. Unemployment increases with the probability measure of getting fired from the job and decreases with the probability of finding a new job while unemployed. As the probability is an increasing function of the effort devoted, this means that if the unemployed devote less search effort, the probability of reemployment will decrease and, hence, the steady state unemployment will rise.

3 The Laissez Faire Equilibrium

In the laissez faire equilibrium, the transfer of market, non-storable goods from the policymaker to the agents is zero for every period. This corresponds to evaluating the economy in $(z_{U,t}, z_{E,t})$ both equal to zero for every period $t$. We are interested in finding the Laissez faire equilibrium to use it as a benchmark. In this way we can compare this equilibrium with other equilibria with positive unemployment insurance to be developed in section 4.

3.1 The Expected Discounted Value of Utility

We are now interested in finding explicitly the value of the discounted utility of the agents in this economy, in terms of fundamentals. Recall that the value of discounted utility of employed and unemployed agents is:

\[
\begin{align*}
V_E &= u(\omega) + \beta [(1 - \phi)V_E + \phi V_U] \\
V_U &= u(0) - C(e) + \beta [P(e)V_H + (1 - P(e))V_U]
\end{align*}
\]

By combining both results, we can obtain explicitly the discounted value of the utility of being employed and unemployed, in terms of the optimal job searching
effort and fundamentals:

\[
V_E = \frac{(1 - \beta(1 - P(e))) [u(\omega)] + \beta \phi [u(0) - C(e)]}{(1 - \beta)(1 - \beta(1 - P(e) - \phi))}
\]

\[
V_U = \frac{(1 - \beta(1 - P(e))) [u(0) - C(e)] + \beta P(e) [u(\omega)]}{(1 - \beta)(1 - \beta(1 - P(e) - \phi))}
\]

### 3.2 Equilibrium Search Effort

The optimal job searching effort of the representative unemployed agent solves the following equation:

\[
e = \arg \max_{e \geq 0} \{(u(0) - C(e)) + \beta [P(e)V_E + (1 - P(e))V_U]\}
\]

The first order condition is:

\[
-\frac{\partial C(e)}{\partial e} [1 - \beta (1 - \phi)] + \frac{\partial P(e)}{\partial e} \beta u(\omega) - (1 - \beta) \beta \frac{\partial P(e)}{\partial e} V_U = 0
\]

with equality if the optimal job search effort is strictly positive. The second order condition is assumed to hold, at least locally\(^1\).

**Proposition 1** When there is no unemployment benefit, the optimal search effort is non-zero.

The proof is available upon request. It basically uses the first order condition together with the fact that the utility and the probability of finding a job are both strictly increasing functions of their respective arguments, and as the wage paid to employed agents (\(\omega\)) is strictly positive. The natural consequence is that the first order condition in the steady state laissez-faire equilibrium holds with strict equality. The optimal job search effort is the one that is implicitly defined by the first order condition.

We next show that the equilibrium level of effort with no insurance is finite.

**Proposition 2** In the laissez-faire steady state equilibrium, the optimal search effort is finite.

**Proof.** Suppose that when there is no unemployment benefit, the optimal effort is infinite. This is, the first order condition should verified when \(e \to +\infty\).

\[
\frac{\partial C(e)}{\partial e} [1 - \beta (1 - \phi)] + \frac{\partial P(e)}{\partial e} \beta u(\omega) - (1 - \beta) \beta \frac{\partial P(e)}{\partial e} V_U(e) = 0
\]

where \(V_U(e) = \left\{ \frac{[1 - \beta (1 - \phi)] [u(0) - C(e)] + \beta P(e) u(\omega)}{(1 - \beta)(1 + P(e) - \beta (1 - \phi))} \right\}\). If we evaluate the discounted expected value of being unemployed, \((V_U(e))\), when the effort is infinite, takes infinitely negative value

\[
\lim_{e \to \infty} V_U(e) = \frac{u(0) - \lim_{e \to \infty} C(e) + \beta u(\omega)}{(1 - \beta)} = -\infty
\]

\(^1\)This of course implies further restrictions on the cost function, the probability function and the utility function. Although we could have assumed these restrictions explicitly, we prefer just to leave this issue aside since it is not central to our focus.
We are making use of the fact that the probability of finding a job if unemployed when the effort is infinite is one (\(\lim_{e \to \infty} P(e) = 1\)). Then:

\[-\lim_{e \to \infty} \frac{\partial C(e)}{\partial e} [1 - \beta (1 - \phi)] + \lim_{e \to \infty} \frac{\partial P(e)}{\partial e} \beta u(\omega) = -\beta \lim_{e \to \infty} \frac{\partial P(e)}{\partial e} [u(0) - \lim_{e \to \infty} C(e)] + \beta u(\omega) = 0\]

As the probability is a strictly increasing and bounded function, when the search effort is infinite, the derivative of the probability with respect to the effort can be shown to be zero. On the other hand, the cost of job searching, when the job searching effort is infinite is also infinite. Then when the job searching effort is infinite, the first order condition does not hold, as it is negative.

4 The Unemployment Benefit

Consider now the design of an optimal unemployment insurance scheme. This scheme, on every date, transfers an amount \(z_E\) of consumption good to any employed agent and will transfer \(z_U\) of consumption good to any unemployed agent. The intertemporal discounted value of the utility with this unemployment insurance is:

\[U_i(f_{c_i,t}; z_{i,t}; e_{i,t}) = \sum_{t=0}^{\infty} \beta^t [u(c_{i,t}) - C(e_{i,t})]\]

where consumption is equal to the sum of the wage paid by the firm (only if employed) and the transfer of the non-storable market goods. We consider a stationary unemployment benefit scheme. This means that the transfer to the unemployed and the employed are both constant through time. Thus:

\[z_{i,t} = \begin{cases} z_E & \text{if the agent } i \text{ is employed at date } t \\ z_U & \text{if the agent } i \text{ is unemployed at date } t \end{cases}\]

We restrict our possible policy analysis to those which does which not decrease consumption if the agent is unemployed (non negative \(z_U\)), because agents cannot consume negative values. The same restriction for employed agents imposes that the transfer if unemployed plus the equilibrium wage cannot be negative. Therefore, we are interested in imposing the first restriction to the unemployment insurance scheme, that is \(z_U \geq 0\) and \(z_E + \omega \geq 0\). For reasons that will be clear in the following sections, we name the value of the transfer if unemployed \((z_U)\) as the unemployment benefit and the negative value of the transfer if employed \((-z_E)\) as the employment tax. The conjerture is that the optimal unemployment insurance will be conformed on positive transfers if unemployed (positive \(z_U\)) and negative transfers if employed (negative \(z_E\)). Notice that as job search effort is unobservable for the principal, the unemployment benefit cannot be contingent on this variable. However, it can be contingent on the agent’s employment state in the present date (employed or unemployed), which is assumed to be perfectly verifiable. This second restriction generates a new job searching effort if unemployed which generates a new steady state distribution of agents. The next constraint is that, given this new distribution of agents
in steady state, the unemployment insurance scheme is a system of transfers (positive and negative) so that it conforms a self-financed policy for every period \( t \):

\[
\int_0^1 \tilde{z}_{i,t} \; di = 0, \forall \; t. \]

Notice that these two restrictions are intrinsically related. The unemployment insurance scheme contingent on the employment state affects the search effort level, affecting the probability of reemployment if unemployed and, hence, affecting the distribution of agents (unemployment and employment) in steady state. Simultaneously, the steady state distribution of agents determines the value of the unemployment benefit that can be transferred between employed and unemployed agents in each period.

A final desired (but not strictly necessary) restriction to the unemployment insurance scheme is that it is welfare enhancing for every agent in the economy in every period of time. Hence, the unemployment benefit is a Pareto improving policy. Shall this property hold for at least some unemployment insurance self-financed contract, then the existence of a positive optimal self-financed unemployment insurance is trivial. If there is an unemployment insurance program where both employed and unemployed agents are better off, then this contract is preferrable to the laissez faire equilibrium. As will be seen in the following sections, this property will not generally be true. Therefore to show the existence of an optimal (positive) unemployment insurance which is self-financed becomes a non-trivial task.

The optimal unemployment benefit maximizes the expected value of social utility, given the condition that is is a self-financed policy and given the change in the incentives to search effort it generates. The optimal unemployment benefit is then a system of positive and negative transfers, so that:

\[
\max_{\tilde{z}} \int_0^1 w_i \sum_{t=0}^{\infty} \beta^t \left[ u(c_{i,t}) - C(e_{i,t}) \right] \; di
\]

s.t.: \( z_{i,t} = z_U \), if agent \( i \) is unemployed at time \( t \).

s.t.: \( z_{i,t} = z_E \), if agent \( i \) is employed at time \( t \).

s.t.: \( c_{i,t} \geq 0, \forall \; i, \forall \; t \)

s.t.: \( \int_0^1 z_{i,t} \; di = 0, \forall \; t \)

s.t.: \( e_{i,t} = e(z_{i,t}), \forall \; i, \forall \; t \)

We are interested in demonstrating the existence and the characterization of an unemployment benefit scheme which maximizes the discounted expected value of social utility subject to the following conditions:

- It makes every agent consume non-negatively in every period.
- It is a self-financed policy.
- It takes into account the effect of the unemployment benefit scheme over incentives to job search.

It is interesting to check whether this unemployment insurance program is accepted voluntarily by every agent (employed or unemployed) in the economy, so that no compulsiveness is needed to take it into practice. If the optimal unemployment insurance is a voluntary accepted contract, the policymaker who provides the unemployment benefit could either be the government or alternatively, a competitive benefit maximizing private firm.
4.1 Distribution in Steady State

The agent dynamics have the same functional equations as the laissez-faire equilibrium, with the only change that the provision of an unemployment benefit will potentially affect the optimal job searching effort, affecting the steady state distribution. Hence, the agent dynamics is characterized by the same system of equations:

\[
\begin{align*}
U_t + E_t &= 1, \quad \forall \ t \\
E_{t+1} &= P(e)U_t + (1 - \phi) E_t, \quad \forall \ t \\
U_{t+1} &= (1 - P(e)) U_t + \phi E_t, \quad \forall \ t
\end{align*}
\]

As we are again interested in the steady state equilibrium of this economy, we can reexpress these equations into their well known steady state version. By solving the system, we get to exactly the same expression for the steady state distribution of agents:

\[
\begin{align*}
U &= \frac{\phi}{P(e) + \phi} \\
E &= \frac{P(e)}{P(e) + \phi}
\end{align*}
\]

Recall that this distribution of agents is different from the laissez-faire steady state distribution of agents as long as the optimal search efforts in both equilibria differ. Trivially, if the provision of the optimal unemployment insurance generates lower job search effort if unemployed, then, the probability of reemployment decreases, so the steady state equilibrium unemployment increases and employment decreases. Moreover, it is trivial to show that, as the probability of receiving a job offer if unemployed when there is no job effort is zero, then zero searching effort is inconsistent with a steady state equilibrium where both employment and unemployment are intertemporally positive constants. This condition will be used in the subsequent results.

4.2 The Self-Finance Condition

Recall that the condition of self-financing of the unemployment benefit system is:

\[
\int_0^1 z_{i,t} \, di = 0, \quad \forall \ t
\]

This imposes that the net value of the transfer given from the policymaker to the society, in every period, is null. In terms of our steady state distribution, there are two types of representative agents, the employed and the unemployed. Therefore, this self-finance condition can be reexpressed as the following steady state condition:

\[
z_E E + z_U U = 0
\]

Replacing the employment and unemployment by known values, we can rewrite it as:

\[
z_E \left( \frac{P(e)}{P(e) + \phi} \right) + z_U \left( \frac{\phi}{P(e) + \phi} \right) = 0
\]
Since the firing probability (if employed) is positive, we obtain the short form of the self-finance condition:

\[ z_E P(e) + z_U \phi = 0 \]

Notice that the autofinance condition is a function of the benefit, the optimal search effort and the probability of firing, which is assumed to be a constant fundamental. In the following section, we prove that the optimal search effort is a function of the unemployment benefit paid (if employed and if unemployed). Hence, we can reexpress this self-finance condition into a relationship between the unemployment benefit transfers, given the parameters.

Second, notice that as the probability of reemployment is nonnegative and the firing probability is positive, if there is a positive transfer if unemployed (positive \( z_U \)), this means necessarily that the transfer if employed must be negative (negative \( z_E \)). This gives us a very logical condition: in order to finance a positive unemployment benefit with an autofinance condition, there will necessarily be an employment tax.

Another feature of the self-financed condition is that if the unemployment insurance is such that the effort is zero, then the probability of reemployment is also zero. So the unemployment benefit that could be provided is zero, no matter the value of the unemployment tax. Recall that if the effort is zero, then the steady state distribution of agents collapses to full unemployment. This means that no unemployment benefit is given to every agent in the economy (unemployed ones) and any employment tax is charged upon nobody (employed ones). That is, when the job searching effort is zero, it generates a steady state distribution where nobody is employed, so the (negative) transfer if employed lacks relevance.

### 4.3 The Expected Discounted Value of Utility

The expected discounted value if the agent is employed (unemployed), can be respectively expressed as \( V_E \) (\( V_U \)):

\[
V_E = u(\omega + z_E) + \beta [(1 - \phi)V_E + \phi V_U] \\
V_U = u(z_U) - C(e) + \beta \left[ PV_E + (1 - P)V_U \right]
\]

The obvious, explicit difference with Laissez - Faire is that the period utility of each agent includes the unemployment benefit and the employment tax. The less obvious and implicit difference is that when there is an unemployment benefit and an employment tax, the job searching effort also changes. Our conjecture is that when the transfer if unemployed increases and/or when the transfer if employed decreases, then the job searching effort also decreases.

Essentially, taking transfers as given, we can solve the equations in exactly the same way as in the laissez-faire equilibrium, obtaining the value of employed and unemployed as a function of the optimal search effort, parameters and, in this case, the transfers.

\[
V_E = \frac{[1 - \beta (1 - P(e))] [u(\omega + z_E)] + \beta \phi [u(z_U) - C(e)]}{(1 - \beta) (1 + \beta P(e) - \beta (1 - \phi))} \\
V_U = \frac{[1 - \beta (1 - \phi)] [u(z_U) - C(e)] + \beta P(e) [u(\omega + z_E)]}{(1 - \beta) (1 + \beta P(e) - \beta (1 - \phi))}
\]
4.4 The Optimal Search Effort

The optimal job searching effort maximizes the expected discounted value of the utility if unemployed, given the unemployment scheme transfers:

\[ e = \arg \max_{e \geq 0} \{ [u(z_U) - C(e)] + \beta [P(e)V_E(e) + (1 - P(e))V_U(e)] \} \]

Replacing the discounted expected values of utility found in the previous sections:

\[ e = \arg \max_{e \geq 0} \left\{ [u(z_U) - C(e)] + \frac{\beta u(\omega + z_E)P(e) + \beta [(1 - \beta) (1 - P(e)) + \phi \beta] [u(z_U) - C(e)]}{(1 - \beta) (1 + \beta P(e) - \beta (1 - \phi))} \right\} \]

By operating algebraically,

\[ e = \arg \max_{e \geq 0} \left\{ \frac{[1 - \beta (1 - \phi)] [u(z_U) - C(e)] + \beta P(e)u(\omega + z_E)}{(1 - \beta) (1 + \beta P(e) - \beta (1 - \phi))} \right\} \]

For notational purposes, let \( N \) and \( D \) be the numerator and the denominator, respectively, of the previous equation, so:

\[ N = [1 - \beta (1 - \phi)] [u(z_U) - C(e)] + \beta P(e)u(\omega + z_E) \]
\[ D = (1 - \beta) (1 + \beta P(e) - \beta (1 - \phi)) \]

The first order condition of the job searching problem is

\[ -\frac{\partial C(e)}{\partial e} [1 - \beta (1 - \phi)] + \frac{\partial P(e)}{\partial e} \beta u(\omega + z_E) \left[ \frac{1}{D} \right] \frac{N}{D^2} = 0 \]

where the condition can be stated with strict equality only if the job searching effort is nonzero. Notice that the term \( D \) is positive, so the first order condition could be simply expressed as:

\[ -\frac{\partial C(e)}{\partial e} [1 - \beta (1 - \phi)] + \frac{\partial P(e)}{\partial e} \beta u(\omega + z_E) \left[ \frac{1}{D} \right] V_U = 0 \]

where again the condition can be stated with strict equality only if the job searching effort is nonzero, and where \( V_U \) is the expected discounted value of utility if unemployed.

Exactly as in the laissez faire equilibrium, the second order condition to the optimal search effort problem is assumed to hold.

**Proposition 3** The search effort is null if and only if the steady state consumption of unemployed \((z_U)\) agents is equal or higher than the steady state consumption of employed agents \((\omega + z_E)\).
\textbf{Proof.} First, we prove that if $z_E$ and $z_U$ are such that the consumption of unemployed is greater than or equal to the consumption of employed, then the optimal job effort is null. If the consumption of unemployed is greater than or equal to the consumption if employed then the following inequality holds

$$\omega + z_E \geq z_U$$

Hence, as the period utility is an strictly increasing function, the following inequality will also hold

$$u(z_U) \geq u(\omega + z_E)$$

which means that the period utility if employed is less or equal that the period utility of being unemployed. To prove that the optimal job searching effort is zero, it must fulfill the first order condition

$$- \frac{\partial C(0)}{\partial e} [1 - \beta (1 - \phi)] + \beta \frac{\partial P(0)}{\partial e} (u(\omega + z_E) - u(\omega)) - (1 - \beta) \beta \frac{\partial P(0)}{\partial e} V_U(0) \leq 0$$

with the expected discounted value of utility if unemployed taking the following value

$$V_U = \frac{u(z_U)}{(1 - \beta)}$$

replacing the expected discounted value of utility in the first order condition

$$- \frac{\partial C(0)}{\partial e} [1 - \beta (1 - \phi)] + \beta \frac{\partial P(0)}{\partial e} [u(\omega + z_E) - u(z_U)] \leq 0$$

using the fact that the marginal cost of the job searching effort is zero when the effort is also zero

$$\beta \frac{\partial P(0)}{\partial e} [u(\omega + z_E) - u(z_U)] \leq 0$$

as the probability is an increasing function of the job searching effort, the previous equation will be true only if the period utility if unemployed is equal or higher that the period utility if employed. To complete the proof, we must show that if the optimal job effort is zero, then the steady state consumption of unemployed agents must be equal or higher than the steady state consumption if employed. This can be trivially done following the previous proof in the inverse order. 

\textbf{Corollary 4} If with unemployment benefit, the steady state consumption when unemployed is less than the steady state consumption when employed, then zero is not the optimal job searching effort. Hence, the first order condition in the optimal job searching problem can be stated with strict equality.

\textbf{Definition 5} The zero effort region are the unemployment insurance scheme contracts, where the agents perform zero effort.

$$\omega + z_E \leq z_U$$

The zero effort line is the hyperplane of the zero effort region.

$$\omega + z_E = z_U$$
The next step is to compare two economies with small differences in the unemployment benefit scheme. We want to study how incentives to search for a new job if unemployed differ. Recall that when the optimal search effort is non-zero, the first order condition is:

\[
\frac{\partial V_U}{\partial e} = -\frac{\partial C(e)}{\partial e}[1 - \beta(1 - \phi)] + \frac{\partial P(e)}{\partial e}\beta u(\omega + z_E) \left[\frac{1}{D} - (1 - \beta)\beta \frac{\partial P(e)}{\partial e}\right] N \frac{D^2}{D^2} = 0
\]

where \( D \) and \( N \) were defined as follows:

\[
D \equiv (1 - \beta P(e) + \beta(1 + \phi))(1 - \beta) > 0
\]
\[
N \equiv [1 - \beta(1 - \phi)][u(\varepsilon + z_U) - C(e)] + \beta P(e)u(\omega + z_E)
\]

Taking that the second order condition holds with a strictly inequality, we can combine this two results to calculate the effect of marginal changes in the transfer given to employed and unemployed agents, by using the implicit function theorem:

\[
\frac{\partial e(\varepsilon)}{\partial \varepsilon U} = -\left[\frac{(1 - \beta)\beta [1 - \beta(1 - \phi)] \frac{\partial P(e)}{\partial e} \frac{\partial u(\omega + z_E)}{\partial z_U}}{(1 - \beta P(e) + \beta(1 + \phi)^2 (1 - \beta) \frac{\partial e}{\partial z_U}} S.O.C.\right] < 0
\]

\[
\frac{\partial e(\varepsilon)}{\partial z_E} = -\left[\frac{(1 + \beta)(1 - P(e)) + \beta\phi \beta \frac{\partial P(e)}{\partial e} \frac{\partial u(\omega + z_E)}{\partial z_E}}{(1 - \beta P(e) + \beta(1 + \phi)^2 (1 - \beta) \frac{\partial e}{\partial z_E}} S.O.C.\right] > 0
\]

where \( S.O.C. \) is the second derivate of the value of utility if unemployed with respect to the job searching effort. This gives a very intuitive result. Any unemployed agent decreases his effort if there is more payment if unemployed, because the consequences of remaining unemployed are less severe. In the same way, if there is more payment if employed, or similarly, if there is less tax if employed, there is a bigger price for being employed, resulting into a higher effort in job searching activity.

We can summarize our findings by stating that job searching effort if unemployed (in any period \( t \)), is an increasing function of the unemployment benefit paid when unemployed and a decresing function of the employment benefit paid when employed (i.e. increasing function of the employment tax charged if employed).

### 4.5 Feasible Contracts

In this section we deduce a relationship between \( z_U \) and \( z_E \) that defines the feasible contracts for the policymaker. The feasible unemployment benefits are such that generate a self-financed policy that respects the best response equation.

The feasible condition can be obtained by inserting the optimal effort function \( e(z_E, z_U) \) into the self-financed condition \( z_E P(e) + z_U \phi = 0 \). We obtain
the following implicit continuous relationship between \( z_U \) and \( z_E \), that defines feasible contracts:

\[
F(z_E, z_U) = z_E P(e(z_E, z_U)) + z_U \phi = 0
\]

**Definition 6** The feasibility curve contains the unemployment insurance contracts that fulfill the condition of self-financement of the unemployment insurance benefit while taking into account that the optimal search effort if unemployed. Implicitly, the feasible contracts are the vector of transfers if unemployed and employed that solve the following implicit equation.

\[
F(z_E, z_U) = z_E P(e(z_E, z_U)) + z_U \phi = 0
\]

As the solution to the optimal search effort is continuous, as the distribution of agents is continuous and as the self-financed condition is a continuous equation, notice that the feasibility curve is a continuous curve.

We are interested in unemployment insurance policies that cannot decrease further unemployed agent’s consumption, so we impose that the unemployment benefit paid to the unemployed is nonegative. Another way of imposing this restriction is to say that agents cannot consume negative on any period: \( z_U \geq 0 \). By imposing that the agents cannot consume negative on any period on employed agents, we get the following restriction:

\[
z_E + \omega \geq 0
\]

Given the labor income of the employed (\( \omega \)), the positive effort restriction restricts the possible locus of the optimal unemployment insurance system contracts. Clearly, the feasible curve includes the laissez faire situation (\( z_E = z_U = 0 \)), or, in other words, the laissez faire situation is feasible. It is less trivial to show the other extreme of this curve.

**Proposition 7** The feasible curve cuts the zero effort line only at the point \((z_E, z_U) = (-\omega, 0)\)

**Proof.** Recall that the zero effort line is, by definition, represented by the equation

\[
\omega + z_E = z_U
\]

and, by construction, any point in the zero effort line induces null optimal search effort. Recall that null search effort level generates a null probability of receiving a job offer if unemployed. By inserting these results in the feasible curve, we obtain:

\[
F(z_E, z_U) = z_E, 0 + z_U \phi = z_U \phi = 0
\]

as the probability of firing is not zero, the transfer if unemployed must be zero: \( z_U = 0 \). Reinserting this condition in the zero effort line we deduce that the transfer to employed agents must be \( z_E = -\omega \).

The question is: how can this be self-financed? How can the policymaker charge a positive tax to the unemployed and distribute nothing to unemployed
and still be a self-financing policy? The answer is quite simple. The policy is charging a positive tax to a measure zero of agents, because nobody is employed. Notice that the after tax consumption of the employed agents is null, and so is the consumption if unemployed. As we proved before, if the after tax consumption for employed and unemployed are the same, the unemployed has no motivation for searching for a job. Therefore, the search effort and the probability of getting a job are both zero. This generates a steady state equilibrium where there are no employed agents, only unemployed. So, basically, the employment tax of $-\omega$ is being charged to nobody. This explains how this positive tax without unemployment subsidy is self financed.

**Lemma 8** Given that $z_E$ is zero, then the feasible curve implies that $z_U$ is zero.

**Proof.** If we insert $z_E = 0$ in the feasible curve $F(0, z_U) = z_U \phi = 0$. Therefore, as $\phi$ is not zero, $z_U$ must be equal to zero. ■

**Proposition 9** Given that there is no transfer if unemployed ($z_U = 0$), then the feasible curve implies that the transfer if employed ($z_E$) can be either zero or $-\omega$. This is, if there is no unemployment benefit, then feasibility means that there is either no employment tax or an employment tax equal to $\omega^2$.

**Proof.** If we insert $z_U = 0$ in the feasible curve equation:

$$F(z_E, z_U) = z_E P(e(z_E, z_U)) + 0 \phi = z_E P(e(z_E, 0)) = 0$$

There are two solutions to this equation. The first one is that the transfer if employed is zero, $z_E = 0$, and the second one is that $z_E$ such that the probability of finding a job if unemployed is zero, $P(e(z_E, 0)) = 0$. This can only be true if the search effort when unemployed is zero. This means that the transfer if employed belongs to the no effort region when there is no transfer if unemployed ($z_U = 0$). This is true when the transfer if employed is such that:

$$-\omega \geq z_E$$

But values of employment tax of less than the wage are discarded because they generate negative consumption to unemployed. ■

Applying the implicit function theorem to the curve equation we obtain the marginal change in the feasible transfer to unemployed agents if there is an infinitesimal change in the transfer to employed agents:

$$\frac{\partial z_U}{\partial (-z_E)} = -\frac{\partial F}{\partial z_E} \frac{\partial z_U}{\partial F} = \left[ \frac{P(e(z_E, z_U)) + z_E \frac{\partial P(e)}{\partial e} \frac{\partial e(z_E, z_U)}{\partial z_U}}{z_E \frac{\partial P(e)}{\partial e} \frac{\partial e(z_E, z_U)}{\partial z_U} + \phi} \right]$$

Clearly the implicit function theorem holds since the denominator cannot be zero. The probability of firing is strictly positive, the transfer to employed is non-positive, the effort is strictly decresing in the unemployment benefit paid and the probability of job searching is an increasing function of the effort made. The expression above shows the slope of the curve in the plane $(z_E, z_U)$. The next proposition characterizes this slope at the extreme points.

---

2 Although employment tax values greater than $\omega$ would also belong to the feasible curve, they imply negative consumption for employed, which is forbidden.
Proposition 10  When there is no unemployment benefit and no employment tax \((z_E = z_U = 0)\), then the slope of the curve is strictly positive:

\[
\frac{\partial z_U}{\partial (-z_E)}(0) > 0
\]

Proof.  Trivially, in the laissez-faire equilibrium, we have already proved that the optimal search effort is positive, so:

\[
\frac{\partial z_U}{\partial (-z_E)}(0) = \frac{P(e(0,0))}{\phi}
\]

The numerator and the denominator are both positive numbers, showing the result.

We can summarize the findings in this section with the following schematic drawing in the \((z_U, -z_E)\) axis:

As we have shown, the feasibility curve starts at the origin \((-z_E, z_U) = (0, 0)\) representing the laissez-faire with positive slope and it ends at the point where the zero effort line intercepts the \(z_U = 0\) axis, which is \((-z_E, z_U) = (\omega, 0)\). Besides from these two points, the feasibility curve does not cut the \(z_U = 0\) axis, the \(z_E = 0\) axis or the zero effort line. Then, as the feasibility curve approaches the vicinity of the point \((\omega, 0)\), the slope of the feasibility curve is necessarily negative, with an absolute value of less than one, since it must not cut the zero effort line near the \((\omega, 0)\):

\[
-1 < \frac{\partial z_U}{\partial (-z_E)}(\omega, 0) < 0
\]

Or equivalently,
Proposition 11  At any point in the feasibility curve different from the origin (the laissez faire equilibrium) there is a non-negative unemployment benefit and there is a positive employment tax. Therefore, any policy that leads to any point in the feasibility curve generates a lower level of job search effort than at the laissez-faire equilibrium.

Proof. Recall that job searching effort if unemployed is a strictly increasing function of both unemployment benefit and employment tax. The origin represents the laissez-faire case, where there is no unemployment benefit and no employment tax. On the contrary, every contract in the feasibility curve which is not the origin has a non-negative unemployment benefit and has a positive employment tax. Hence, by comparing directly the laisse faire case with any other contract in the feasibility curve, we can be sure that the job searching effort has a maximum at the laissez faire equilibrium. Any departure from this point along the feasibility curve means necessarily a lower level of job searching effort.

Clearly then, any point in the feasibility curve different from the origin (the laissez faire equilibrium) represents steady state equilibria where there is more unemployment (and less employment) than in the laissez-faire equilibrium. This is because unemployment (employment) is an increasing (decreasing) function of the job searching effort.

4.6 Voluntary Entering

Suppose that there is a (positive) unemployment benefit that maximizes the discounted utility of the society. One desired condition of this insurance scheme may be that the agents voluntarily accept this scheme in every period of their life and in every possible state (employed or unemployed). This means that on each possible state (employed or unemployed), the discounted expected utility of accepting the unemployment benefit contract is greater than the discounted expected utility of not accepting it.

The employed agents prefer a proposed unemployment benefit scheme to laissez-faire if their value with this proposed contract is greater or equal than the value in laissez faire equilibrium, expressed in terms of fundamentals gives:

\[
\frac{u(z_E + \omega) [1 - \beta (1 - P(e(z_U)))] + \beta \phi (u(z_U) - C(e(z_U)))}{(1 - \beta) (1 + \beta P(e(z_U)) - \beta (1 - \phi))} \geq \frac{u(\omega) [1 - \beta (1 - P(e(0)))] + \beta \phi (u(0) - C(e(0)))}{(1 - \beta) (1 + \beta P(e(0)) - \beta (1 - \phi))}
\]

Simetrically, the unemployed agents prefer a proposed unemployment benefit scheme to laissez-faire if their value with the insurance is greater or equal that the discounted expected value of their utility in laissez faire, expressed as follows:

\[
\frac{[1 - \beta (1 - \phi)] (u(z_U) - C(e(z_U))) + \beta P u(z_E + \omega)}{(1 - \beta) (1 + \beta P(e(z_U)) - \beta (1 - \phi))} \geq \frac{[1 - \beta (1 - \phi)] [u(0) - C(e(0))] + \beta P u(\omega)}{(1 - \beta) (1 + \beta P(e(0)) - \beta (1 - \phi))}
\]
Notice that if the period utility of consuming zero is sufficiently negative, then both conditions will hold for at least some contract \((z_U, z_E)\), as long as consumption for employed and unemployed agents with the proposed contracts is positive. In the case that both conditions hold for a proposed contract, then both employed and unemployed agents will voluntarily accept to participate in the contract. In this case, two interesting features can be stated:

- The policymaker that can provide this proposed unemployment insurance scheme can even be a private firm, since no coercion is necessary to accept the contract.
- The existence of a positive optimal unemployment insurance can be trivially demonstrated. If there is some contract which makes each and every agent better off, then there must be some self-financed contract which maximizes society’s welfare, which is different from laissez faire.

It is intuitively clear how a self financed unemployment insurance contract instantly benefits unemployed agents. However, how could an employed agent accept such a self-financed contract, such that he employed transfers goods to the unemployed? The answer is simple. The employed faces a positive probability of being fired from his job next period. If he finally is actually fired, he would suffer a big period disutility from not consuming while unemployed. However, it is true that the parameter conditions for which both inequalities hold for any proposed (positive) unemployment insurance system are very restrictive. In general, we can say that the voluntary entering conditions do not necessarily hold. If any of these (or both of them) do not hold for some proposed unemployment insurance contract, then it is not voluntary accepted. Without making stronger assumptions we cannot assure that the conditions for voluntary acceptance do hold. In this case, two other features can be stated:

- The proposed unemployment insurance contract can only be provided by the government and not by a private firm, as coercion is needed for its acceptance.
- The existence of an unemployment insurance contract which is different from the laissez faire equilibrium is not at all trivial, and should be proved.

In the following section, which is the core of the work, the existence of a socially optimal unemployment self-financed insurance program is formally demonstrated. In such a case, even if one (or both) agents did not accept voluntarily the optimal contract, it would be socially worthwhile to impose it using coercitive power, because by imposing the contract, the society as a whole increases its welfare.

As a summary of this section, we characterized the equations that imply the voluntary acceptance of the unemployment scheme. If these conditions do hold, then the proposed unemployment insurance could be provided even by private firms. If any of these conditions do not hold, then the proposed unemployment insurance could only be provided by using coercitive power. If there is an optimal unemployment insurance which is not laissez faire, its imposition is socially worthwhile even if it would not be voluntary accepted by each and every agent in society.
4.7 The policymaker’s problem

In this section we characterize the optimal unemployment insurance and prove its existence formally. The optimal unemployment benefit is such that it maximizes the society’s discounted expected utility subject to a self finance condition and to the voluntary acceptance on the unemployment benefit contract:

\[
\max_Z \int_0^1 w_i \sum_{t=0}^{\infty} \beta^t [u(c_{i,t}) - C(e_{i,t})] \, dt \]
\[
s.t.: z_{i,t} = z_U, \text{ if } i \text{ is unemployed at time } t
\]
\[
s.t.: z_{i,t} = z_E, \text{ if } i \text{ is employed at time } t.
\]
\[
s.t.: z_U \geq 0
\]
\[
s.t.: z_E + \omega \geq 0
\]
\[
s.t.: \int_0^1 z_{i,t} \, dt = 0, \forall t
\]
\[
s.t.: e = e(z_U, z_E)
\]

If this problem is solved by the policymaker assuming that at \( t = 0 \) the economy is in steady state, we can reexpress the previous problem by a simpler problem:

\[
\max_Z \ w_E V_E + w_U V_U \\
\text{s.t.: } z_U \geq 0 \\
\text{s.t.: } z_E + \omega \geq 0 \\
\text{s.t.: } F(z_E, z_U) = 0 \Leftrightarrow z_E = z_E(z_U) \\
\text{s.t.: } e = e(z_U, z_E)
\]

Recall that the expected discounted values of utilities are represented by the following equations:

\[
V_E = \frac{(1 - \beta (1 - P(e))) [u(\omega + z_E)] + \beta \phi [u(z_U) - C(e)]}{[1 - \beta (1 - P(e))] [1 - \beta (1 - \phi)] - \beta^2 P(e) \phi}
\]

\[
V_U = \frac{[w_E (1 - \beta) + (w_U + w_E) \beta P(e)] + [w_E \beta \phi + w_U [1 - \beta (1 - \phi)] u(z_U) - C(e)]}{(1 - \beta) (1 + \beta P(e) - \beta (1 - \phi)) u(z_U) - C(e)}
\]

After calculations it can be shown that:

\[
w_E V_E + w_U V_U = \left( \frac{w_E (1 - \beta) + (w_U + w_E) \beta P(e)}{1 - \beta (1 + \beta P(e) - \beta (1 - \phi))} u(z_E + \omega) + \frac{w_E \beta \phi + w_U [1 - \beta (1 - \phi)]}{1 - \beta (1 + \beta P(e) - \beta (1 - \phi))} u(z_U) - C(e) \right)
\]

Inserting the feasible condition, \( F(z_E, z_U) = 0 \), or equivalently, \( z_E = z_E(z_U) \), and by inserting the incentive compatibility constraint \( (e = e(z_U, z_E(z_U))) \) we can rewrite the social value of utility only in terms of the transfer if unemployed \( (z_U) \).

\[
w_E V_E + w_U V_U = \Upsilon_1(z_U) + \Upsilon_2(z_U)
\]

where \( \Upsilon_1(z_U) \) and \( \Upsilon_2(z_U) \) are defined as follows:

\[
\Upsilon_1(z_U) = \frac{w_E (1 - \beta) + (w_U + w_E) \beta P(e(z_U))}{1 - \beta (1 + \beta P(e(z_U)) - \beta (1 - \phi))} u(z_U(z_U) + \omega)]
\]

\[
\Upsilon_2(z_U) = \frac{w_E \beta \phi + w_U [1 - \beta (1 - \phi)]}{1 - \beta (1 + \beta P(e(z_U)) - \beta (1 - \phi))} u(z_U) - C(e(z_U))]
\]
Proposition 12: The laissez-faire contract \((z_U, z_E) = (0,0)\) is not socially optimal.

Proof. Suppose that \(z_U\) equal to zero is optimal. If this is true, the first order condition implies:

\[
\frac{\partial \Upsilon_1(0,0)}{\partial z_U} + \frac{\partial \Upsilon_2(0,0)}{\partial z_U} = 0
\]

Remember that \(e(z_U)\) is bounded, so \(e(0) < +\infty\) and \(P(e(0)) < 1\). Recall also that if \((z_U, z_E) = (0,0)\), then \(\frac{\partial z_U}{\partial (-z_E)} > 0\), (so \(\frac{\partial z_U}{\partial z_E} < 0\)) and \(\frac{\partial e(0)}{\partial z_E} < 0\).
Consider the expression for \( \frac{\partial Y_1(0,0)}{\partial z} \). This is clearly a product and addition of bounded values, so it is bounded: \(-\infty < \frac{\partial Y_1(0,0)}{\partial z_U} < +\infty\). For \( \frac{\partial Y_2(0,0)}{\partial z} \), as the job searching effort is bounded in the laissez faire equilibrium and the marginal period utility of consumption is infinite if the agent consumes zero then \( \frac{\partial Y_2(0,0)}{\partial z} = +\infty \). Combining both results we have

\[
\frac{\partial Y_1(0,0)}{\partial z_U} + \frac{\partial Y_2(0,0)}{\partial z_U} = +\infty
\]

which means that the first order condition does not hold, showing that no unemployment benefit is not socially optimal.

The intuition behind the result is straightforward. In the laissez faire equilibrium, the unemployed agent does not consume, giving him an infinite period marginal utility of consumption. An infinitesimal increase in the unemployed agent’s period consumption makes him consume positively, so his period utility increases infinitely, increasing infinitely society’s welfare. But to provide to the unemployed workers with an infinitesimal increase of consumption we must decrease the employed consumption, reducing their period utility. The clue is that as the employed agents were consuming positively (\( \omega \)), their decrease in period utility was less than infinite.

Joining both results, we deduce that the society increases its welfare by this change.

**Proposition 13** The contract \((z_U, z_E) = (0, -\omega)\) is not socially optimal.

**Proof.** If \((0, -\omega)\) were optimal, the first order condition would hold, evaluated at this point. Thus

\[
\frac{\partial Y_1(0,-\omega)}{\partial z_U} + \frac{\partial Y_2(0,-\omega)}{\partial z_U} = 0
\]

Recall that if \((z_U, z_E) = (0, -\omega)\), then \(\frac{\partial z_E}{\partial z_U} \in (0,1)\) and \(z_E + \omega = z_U = 0\), so there is zero searching effort. Consider again the expression for \( \frac{\partial Y_1(z_U, z_E(z_U))}{\partial z_U} \) evaluated at the point. By the fact that \( \frac{\partial u(0)}{\partial c} = +\infty \), hence \( \frac{\partial Y_1(0,-\omega)}{\partial z_U} = +\infty \). Now take \( \frac{\partial Y_2(z_U, z_E(z_U))}{\partial z_U} \) evaluated at \((0, -\omega)\). As we assumed that the marginal cost of job search effort is zero when the effort is zero, and using that the period marginal utility of consuming zero is infinite, we also get \( \frac{\partial Y_2(0,-\omega)}{\partial z_U} = +\infty \). Therefore, by joining both results:

\[
\frac{\partial Y_1(0,-\omega)}{\partial z_U} + \frac{\partial Y_2(0,-\omega)}{\partial z_U} = +\infty
\]

which means that the first order condition does not hold, so providing this contract is also not socially optimal.

\[\text{---}
\]

\[3\] Recall that in this situation, the unemployed have an optimal search effort equal to zero, and therefore, the probability of obtaining a job offer if unemployed is null.
The intuition behind this result is also straightforward. In this equilibrium, both employed and unemployed are consuming zero. So, both type of agents would increase their period utility infinitely if their consumption increased infinitesimally. By what has been proved before at the point \((0, -\omega)\), it is feasible to reduce the employment tax and to increase the unemployment benefit paid, increasing both consumptions simultaneously. This movement would increase social utility infinitely.

Both results contain the main policy implication. The optimal unemployment insurance (if it exists) is one of the contracts of the continuous feasibility curve excluding laissez-faire and \((0, -\omega)\). As the social utility function is continuous and as the feasibility curve is continuous and closed set of (feasible) contracts, the Weierstrass theorem tells us that one of the feasible contracts must achieve a maximum social utility. This contract that maximises social utility is the optimal unemployment insurance. Certainly, there could be multiple (local) extremes on this curve. In such a case the policymaker should select maxima and discard minima using second order conditions and, afterwards, select the global maximum between the multiple local maxima. This process should result in the global maximum of the social utility along the feasibility curve, which are the optimal unemployment insurance system.

Hence, the Weierstrass theorem tells us that the optimal autofinanced unemployment benefit exists. As we discarded corner solutions, this contract should transfer \(z_U > 0\) to the unemployed agents and should include an employment tax \(z_E\), which is positive and less than \(\omega\). \((z_E \in (0, \omega))\). For the contracts below the zero effort region the optimal search effort is positive, and so is the probability of receiving a job offer if unemployed. Therefore, if the optimal unemployment insurance is below the zero effort region, then the corresponding optimal search effort is positive, and so the probability of receiving a job offer if unemployed is positive. This implies that, with a lower level of optimal job searching effort than laissez-faire, the probability of finding a job if unemployed is less that...
the one under laissez-faire. Therefore, the level of unemployment (employment) with the optimal unemployment insurance system is strictly higher (less) than the prevailing under the laissez-faire equilibrium.

5 Concluding Remarks

In this paper, the problem of the optimal design of an unemployment insurance in the context of a general equilibrium model is considered. A non trivial self-financed optimal unemployment insurance contract was shown to exist under the conditions in our model. The optimal unemployment insurance consists of transfers of market goods between the policymaker and each agent, a transfer that is contingent on the unemployment state of the agent. The unemployed receives a benefit while the employed pays a tax. In this way the insurance contract is self-financed, because the employment tax revenue matches exactly the cost of the unemployment benefit. This optimal unemployment insurance solves the trade-off between insurance and incentives of job searching that arise normally with these contracts: the combination of a positive unemployment benefit with an employment tax generates a decrease in the effort devoted to job search for unemployed agents. As a result of the trade off between incentives and insurance, the effort devoted to search for a new job while unemployed is strictly less than in the laissez-faire equilibrium, resulting in higher unemployment (lower employment) and higher unemployment duration.

We established that the necessary and sufficient conditions so that every agent in the economy (employed or unemployed) accepts voluntarily the optimal unemployment insurance contract are very restrictive. In general, we can state that if every agent accepts the contract voluntarily, then the proposed insurance is also a Pareto improving policy and can be provided by private firms. If either employed or unemployed workers do not accept the optimal unemployment insurance contract, then this should be provided by the government. However, in practice we do not observe private firms providing unemployment insurance. This casual evidence illustrates the fact that the restrictions for the optimal unemployment insurance to be voluntarily accepted by employed and unemployed are unlikely to hold in reality.

The unemployment insurance policy considered is very specific and there are some natural ways in which it could be extended. One extension could be an unemployment insurance policy where the benefit paid to unemployed varies with the length of the unemployment spell and the rate of the tax charged to employed agents varies with the length of the preceding unemployment spell, as in Hopenhayn and Nicolini (1997)'s partial equilibrium model. Given the structure of the model, this is a much harder task and calls for the use of techniques developed by Marcet and Marimon (2000) to deal with history dependent contracts. Another possible dimension is to consider the impact of the unemployment insurance benefit on the working effort performed if employed, besides from considering the impact on job searching effort if unemployed. Notice that the optimal working effort would determine the firing probability if employed, endogenizing the (exogenously assumed) parameter $\phi$ of the model. Another interesting dimension to extend this model would be to incorporate multiple job opportunities in the same economy, fully exploiting the properties of the Lucas and Prescott (1974) islands model, as done in Alvarez and Veracierto (1999).
References


