Capital Humano, Fluctuaciones y Desarrollo

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Rodolfo E. Manuelli

University of Wisconsin-Madison y Universidad T. DiTella

Motivación y Evidencia

- Cuál es la razón última de la riqueza de las naciones?
- Políticas eficientes?
- Grandes diferencias entre países pobres y ricos en:
  - Producto por persona empleada (aproximadamente 20:1)
  - Años de escolaridad de la fuerza de trabajo (12 vs 2.5)
  - Fertilidad (1.8 vs 5.5 + hijos)
  - Esperanza de vida (78 vs. 46 años)

Plan

- El rol del capital humano
  - Consecuencias.
  - Dinámica.
- Incertidumbre y crecimiento:
  - Evidencia
  - Modelos.

Causas Próximas de la Riqueza (Pobreza)

- Nivel del producto por persona empleada?
  - Capital físico por persona.
  - Capital humano por persona.
  - Productividad.

$$\underbrace{Y}_{\text{Producto}} = \underbrace{z}_{\text{Productividad.}} F(\underbrace{K}_{\text{Cap. Físico}}, \underbrace{H}_{\text{Cap. Humano}})$$

- Ejemplo:

$$Y = zK^{\theta}(\bar{h}N)^{1-\theta}$$

- $$y = \frac{Y}{N} = z^{1/(1-\theta)} \left(\frac{K}{Y}\right)^{\theta/(1-\theta)} \bar{h}$$

- Diferencias entre un país pobre y, por ejemplo, USA pueden descomponerse de la siguiente manera:

$$\frac{y_{US}}{y_P} = \left(\frac{z_{US}}{z_P}\right)^{1/(1-\theta)} \left(\frac{(K/Y)_{US}}{(K/Y)_P}\right)^{\theta/(1-\theta)} \frac{\bar{h}_{US}}{\bar{h}_P}$$

- Evidencia: La relación capital-producto a precios internos no varía con el nivel de ingreso.

- Definiendo:

$$\kappa = \frac{rK}{Y}$$

donde r es el precio del capital.

$$\frac{y_{US}}{y_P} = \left(\frac{z_{US}}{z_P}\right)^{1/(1-\theta)} (r_P)^{\theta/(1-\theta)} \frac{\bar{h}_{US}}{\bar{h}_P}$$

- A qué se deben las diferencias?
  - Mayormente a diferencias en la productividad (z).
  - Mayormente a diferencias en el capital humano por trabajador ($\bar{h}$).
- Extensa literatura empírica.
  - América Latina: Víctor Elías
- Estrategia: Medir $\bar{h}$, y “estimar” el z necesario para explicar las diferencias de producto.

Capital Humano: Medición Directa

- Klenow & Rodriguez Clare y Hall & Jones se basan en la regresión de Mincer

$$\bar{h} = e^{\phi s}$$

  - $s =$ años de escolaridad promedio (Barro & Lee)
  - $\phi =$ tasa de retorno a la educacion (Psacharopulous)

- Mincer estima

$$\ln(ingreso) = \beta_0 + \beta_1 s + \beta_2 p + \beta_3 p^2 + error$$

- Diferencias en experiencia (p) son ignoradas (experiencia promedio es similar?)

Capital Humano y Diferencias de Productividad

- Retorno a la educación es 10% ($\phi = .10$)
  - Cerca del promedio mundial.
  - Varía entre 8$+$ y 12-13
- En este caso $\frac{h_{US}}{h_P} = \frac{\text{exp}(.1\times 12)}{\text{exp}(.1\times 2.5)} = \text{exp}(.95) = 2.58$
  - Hall & Jones y Klenow & Rodriguez-Clare, $\frac{h_{US}}{h_P} = 2.2$
  - Suponen un retorno más alto a la educación primaria.

Capital Humano y Diferencias de Productividad (cont.)

- Since $\frac{y_{US}}{y_P} = 20 = \underbrace{\left[\left(\frac{z_{US}}{z_P}\right)^{1/(1-\theta)}\right]}_{5.70} \overbrace{\left[(r_P)^{\theta/(1-\theta)}\right]}^{1.60} \overbrace{\left[\frac{h_{US}}{h_P}\right]}^{2.22}$

- Por lo tanto, $z_P/z_{US} = 0.32$.

- Cuál es el efecto de cambiar z?

- Cuáles son las consecuencias de distorsiones en el sector productor de capital humano?

Midiendo Capital Humano: Ben Porath

$$\max \int_6^R e^{-r(a-6)}[wh(a)(1-n(a))-x(a)]da - x_E$$

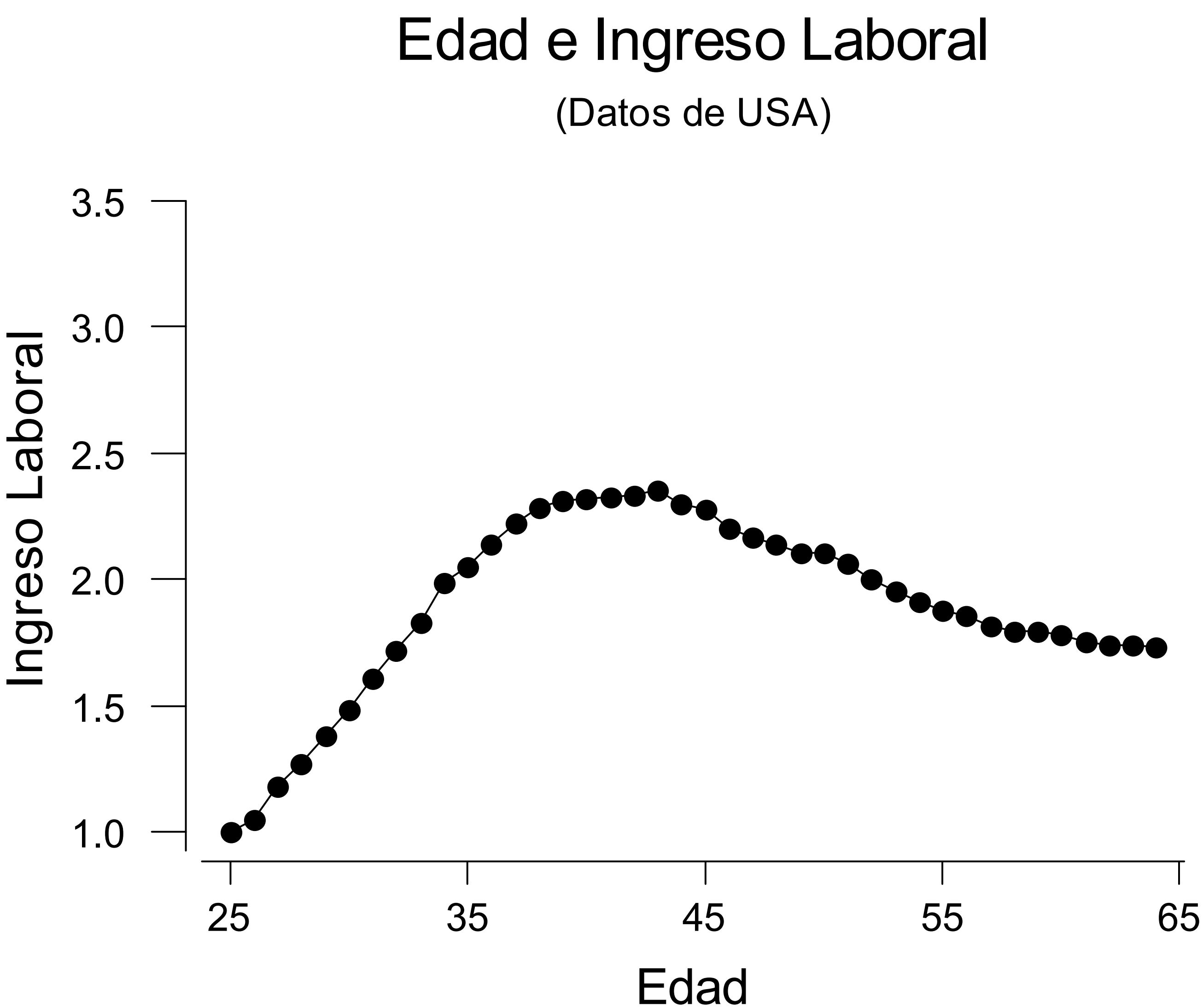
sujeto a

$$\dot{h}(a) = z_h[n(a)h(a)]^{\gamma_1}x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [6, R),$$

y

$$h(6) = h_E = h_B x_E^{\upsilon}$$

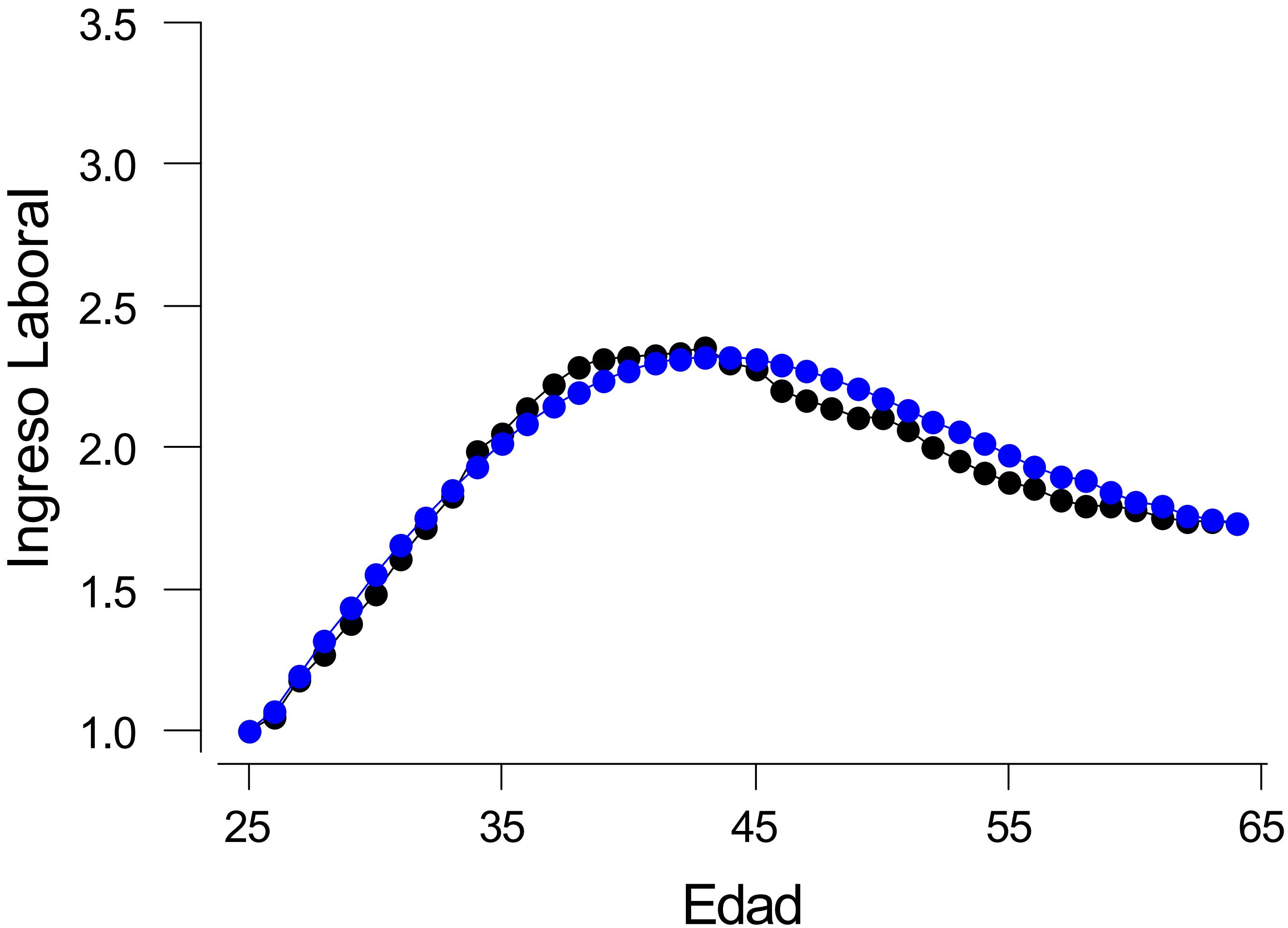
- Evidencia: Cómo estimar los parámetros?
- Resultados: $\gamma_1 = 0.6$, $\gamma_2 = 0.30$, $\gamma = \gamma_1 + \gamma_2$



Edad e Ingreso Laboral
(Datos de USA)
Ingreso Laboral
3.5
3.0
2.5
2.0
1.5
1.0
25
35
45
55
65
Edad

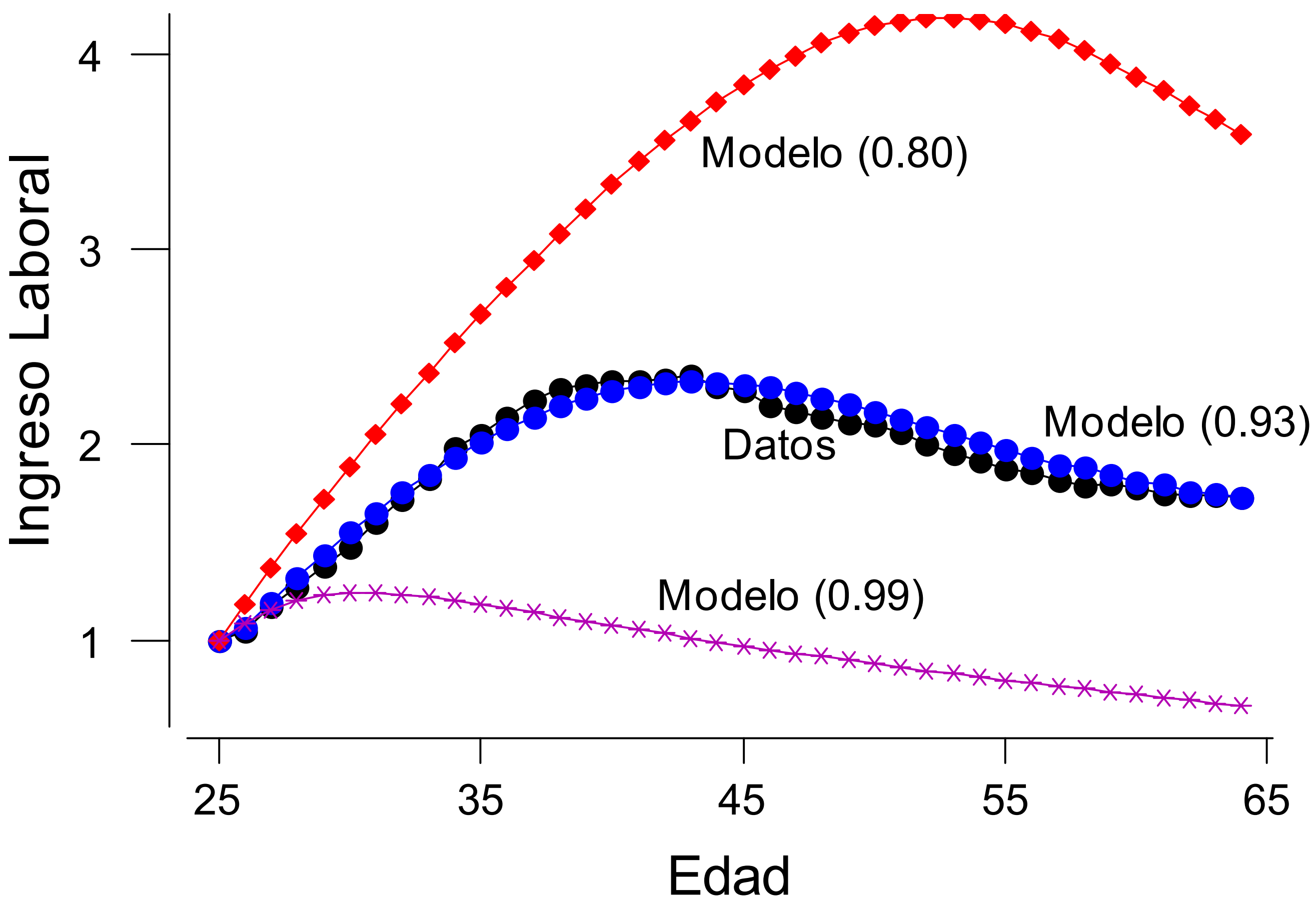
Edad e Ingreso Laboral: Modelo y Evidencia

(Datos de USA)



Edad e Ingreso Laboral: Modelo y Evidencia

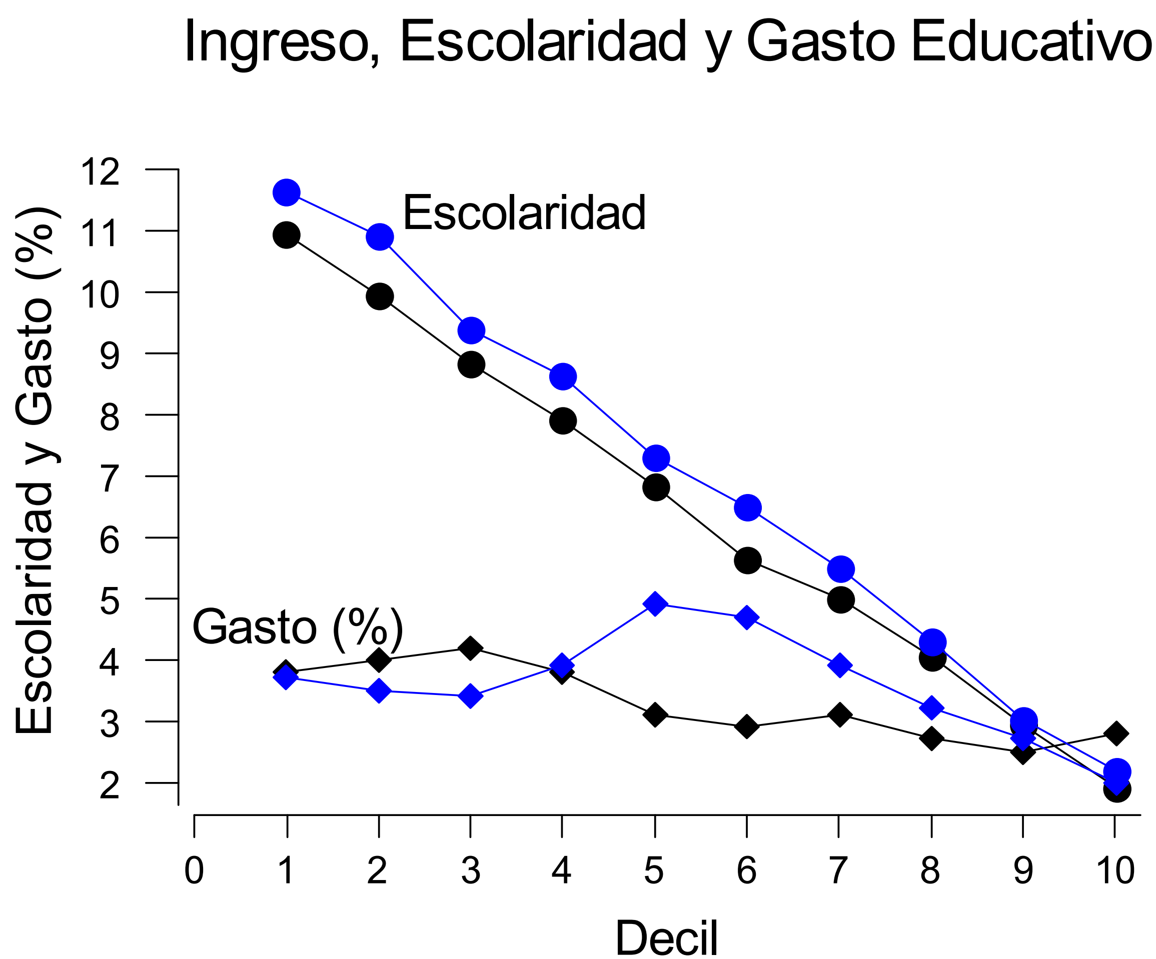
(Datos de USA)



Resultados

- Ejercicio: Elegir la "verdadera" productividad para que el modelo prediga correctamente el nivel de ingreso de cada país.

- Test: En qué medida el modelo puede predecir razonablemente
  - El nivel de escolaridad?
  - El nivel de gasto educativo.

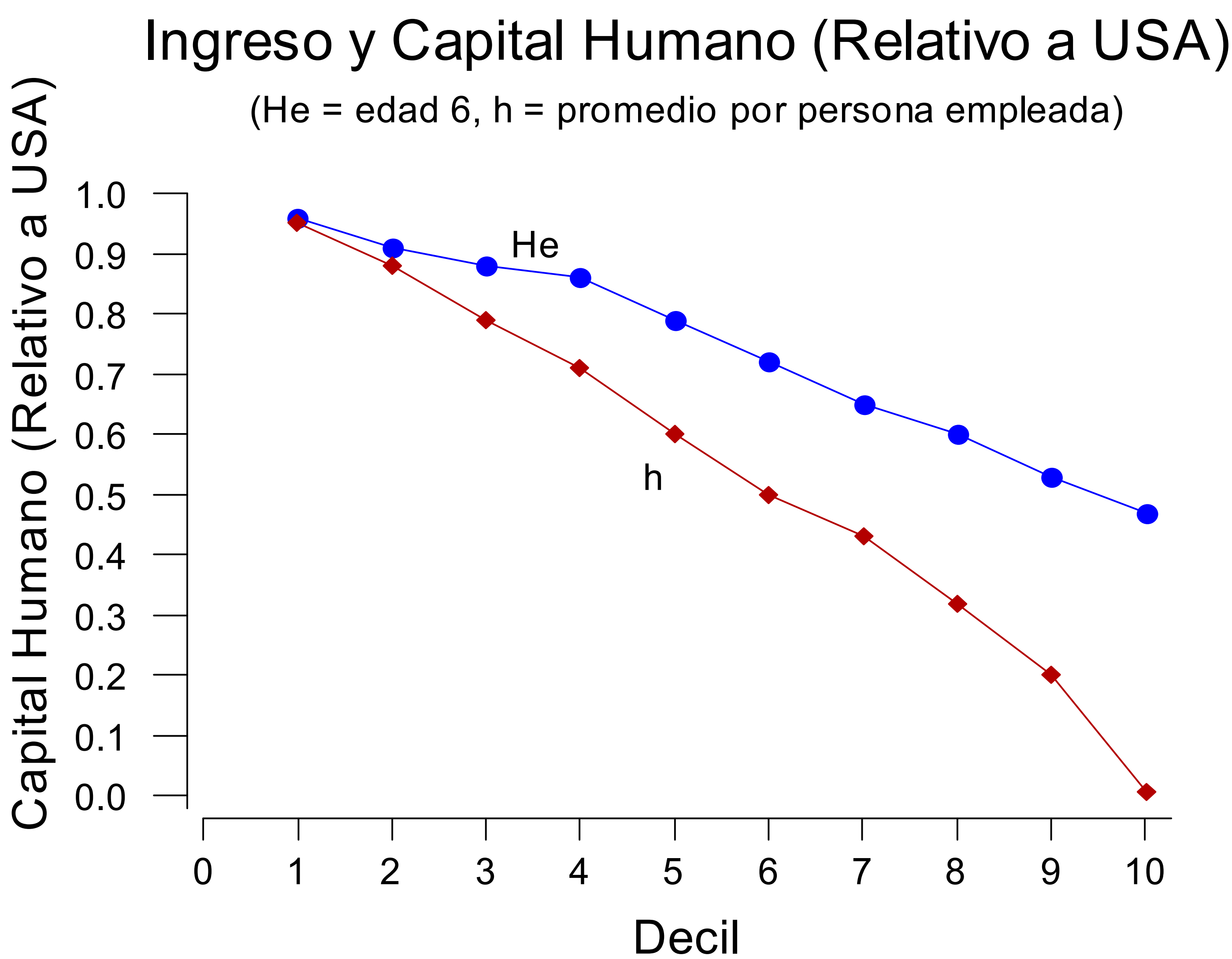


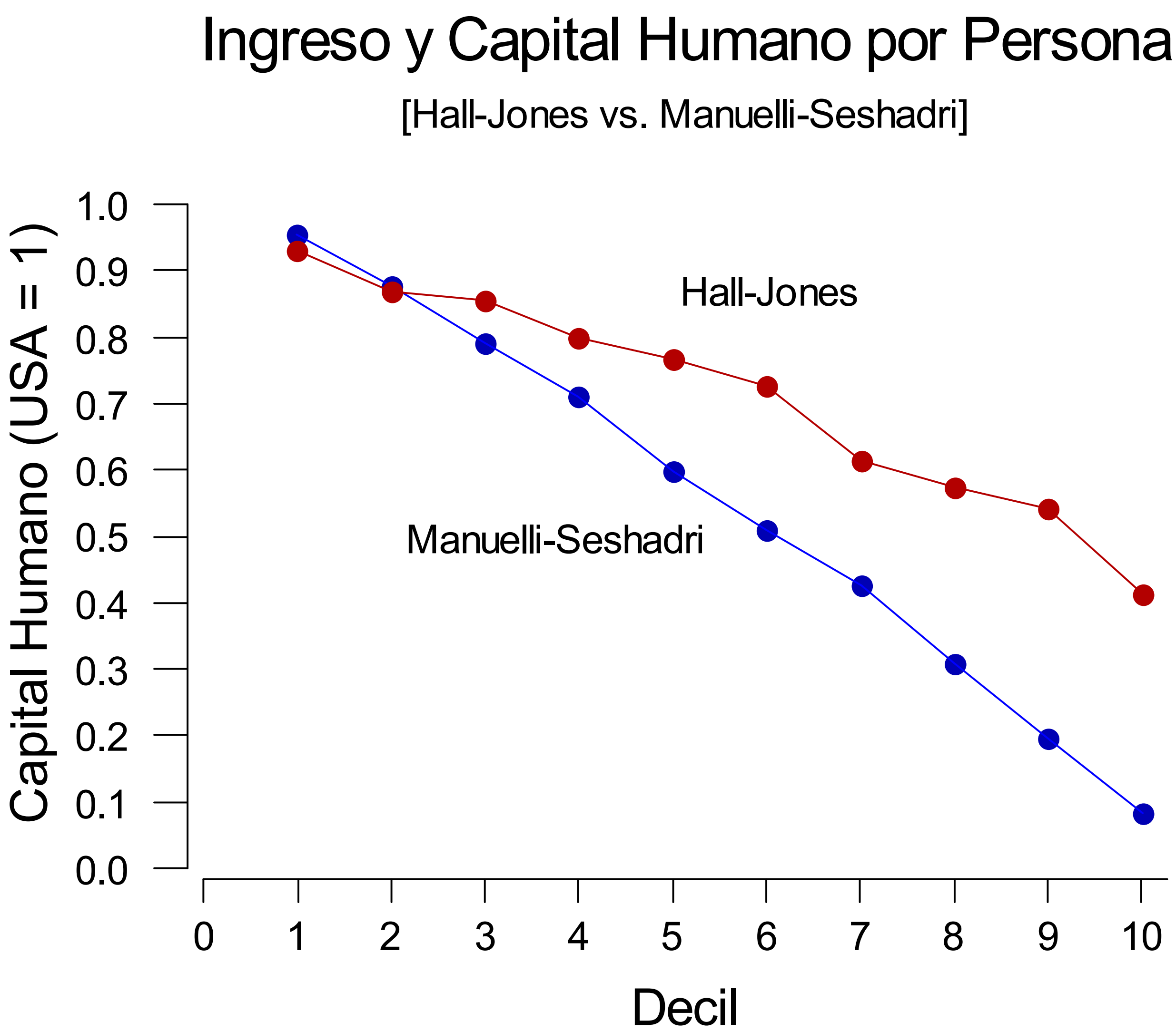
Ingreso, Escolaridad y Gasto Educativo
Escolaridad
Gasto (%)
Escolaridad y Gasto (%)
Decil
12
11
10
9
8
7
6
5
4
3
2
0
1
2
3
4
5
6
7
8
9
10

Resultados (cont.)

- Diferencias de capital humano empiezan antes de la edad escolar.

- Se magnifican por los bajos retornos a la acumulación.


Ingreso y Capital Humano (Relativo a USA)
(He = edad 6, h = promedio por persona empleada)
Capital Humano (Relativo a USA)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
He
h
0
1
2
3
4
5
6
7
8
9
10
Decil


Ingreso y Capital Humano por Persona
[Hall-Jones vs. Manuelli-Seshadri]
Hall-Jones
Manuelli-Seshadri
Capital Humano (USA = 1)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
Decil

Interpretación

- Qué implican estos resultados?
  - Diferencias de productividad no son tan grandes. (30%)
  - Diferencias de capital humano son sustanciales.
  - Reducción de distorsiones:
    * Efecto significativo.
    * Muy de largo plazo.

Dinámica del Crecimiento I

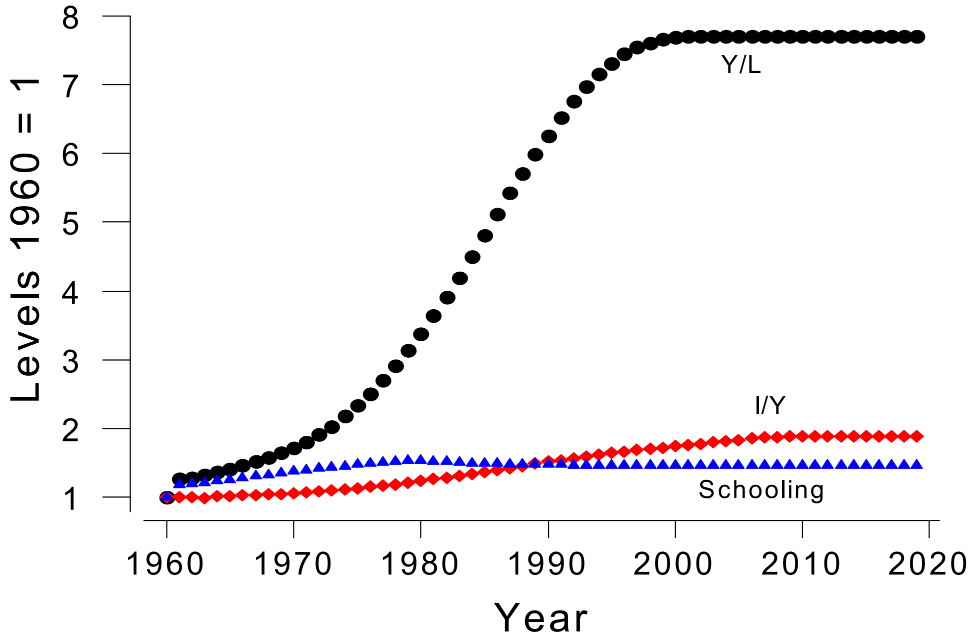
- Aumento de productividad.
- King y Rebelo: Convergencia es rápida.
- Diferencias:
  - Inversión en capital humano aumenta.
  - Effectos heterogéneos:
    * Jóvenes invierten fuertemente
    * No jóvenes invierten poco.
  - El capital humano de la mano de obra va cambiando lentamente.

Una Aplicación: Milagros Económicos.

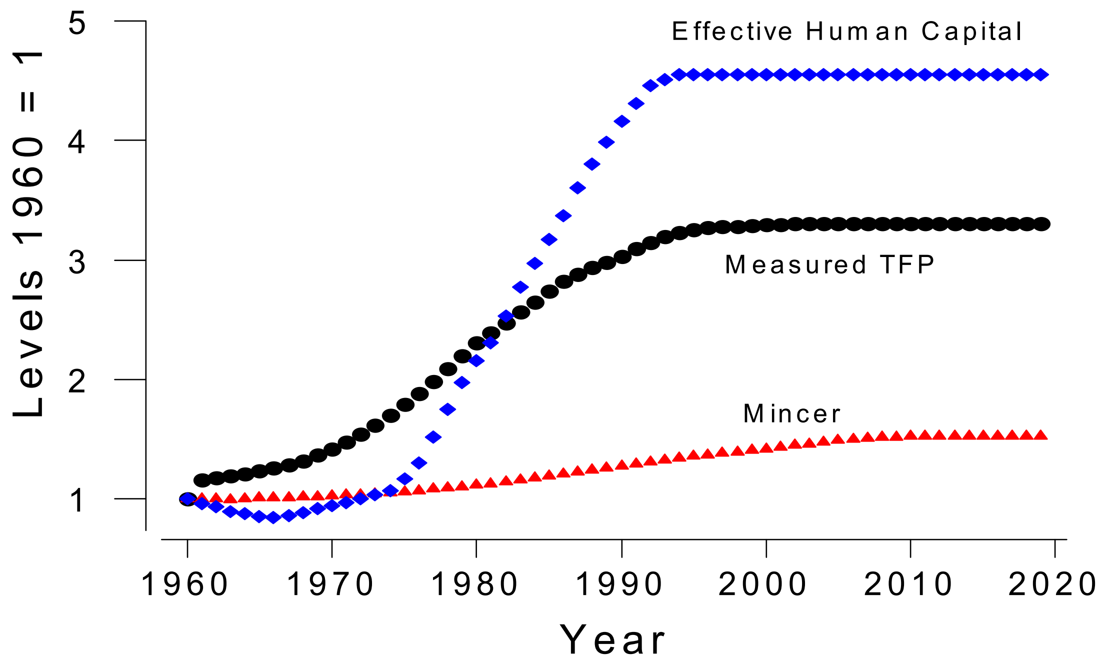
- Usar el modelo para simular el crecimiento de los “tigres asiáticos.”
- Un único aumento de productividad en la década de los 60.
- Ajuste demográfico.

- Países: Corea, Hong Kong, Malasia, Singapur y Taiwan.

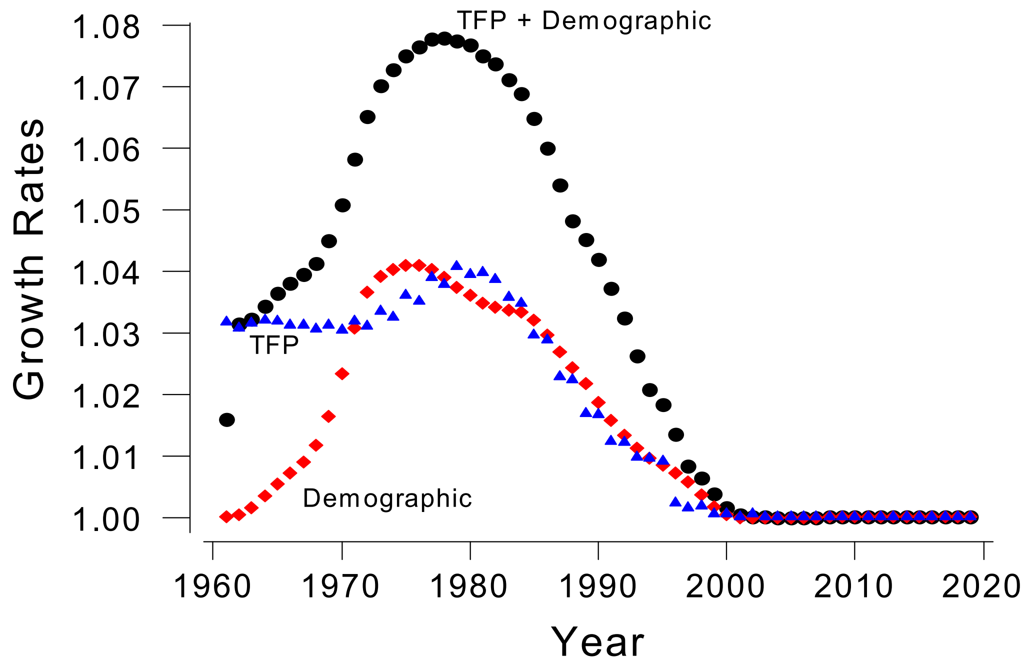
- Evidencia (1960-2000):
  - Producto por trabajador se incrementa siete veces!
  - Escolaridad (fuerza de trabajo) pasa de $3+$ años a 7 años.



Shock: Demográfico y Productividad



Productividad Aparente y Capital Humano (Efectivo y Aparente)



Tasa de Crecimiento: Cambios en Productividad y Estructura Demografica

Capital Humano, Fertilidad y Esperanza de Vida

- Qué predice este modelo sobre la relación entre desarrollo, fertilidad y esperanza de vida?

- Evidencia:
  - Relación inversa entre producto por persona y fertilidad.
  - Relación inversa entre esperanza de vida y fertilidad.
  - Relación inversa entre producto por persona y esperanza de vida.

Teoría

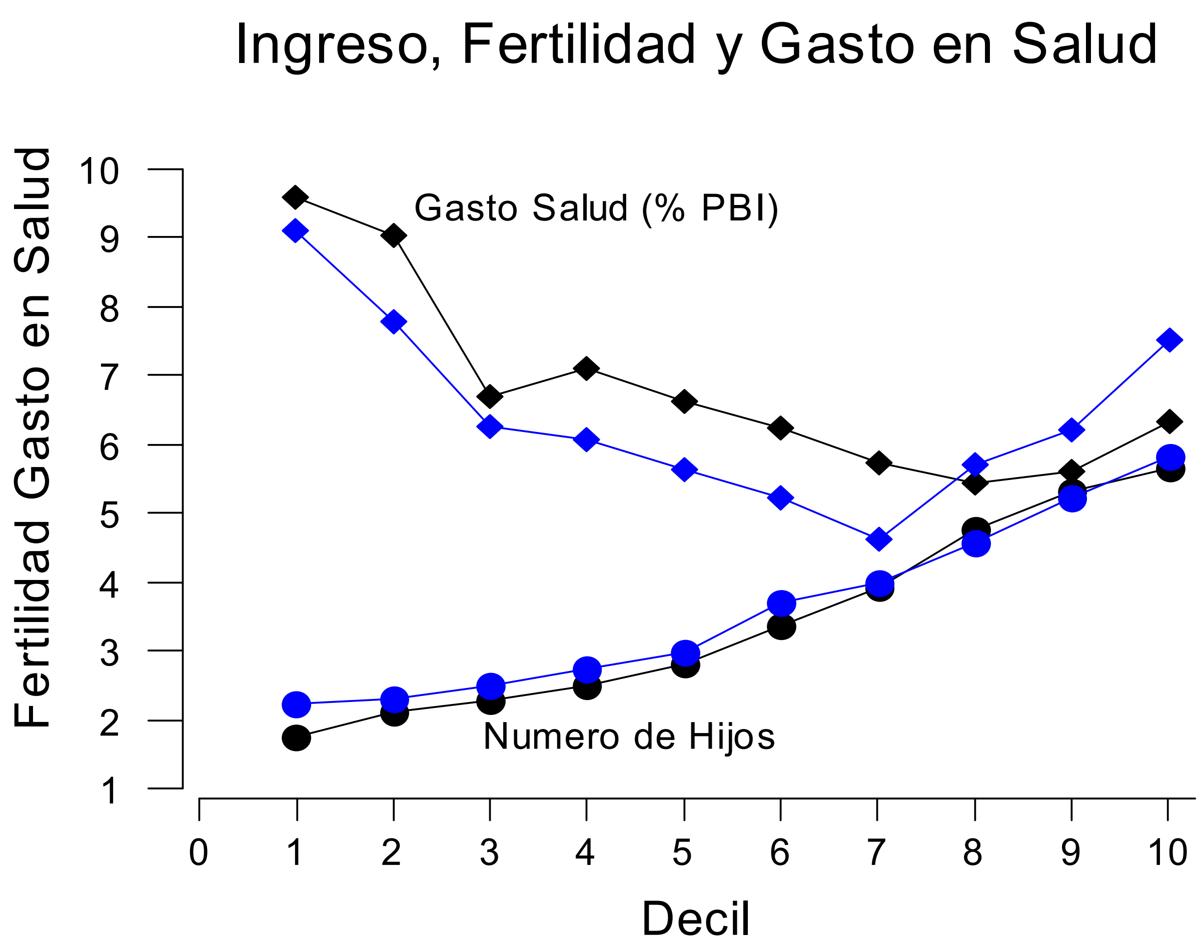
- Basado en Barro-Becker:

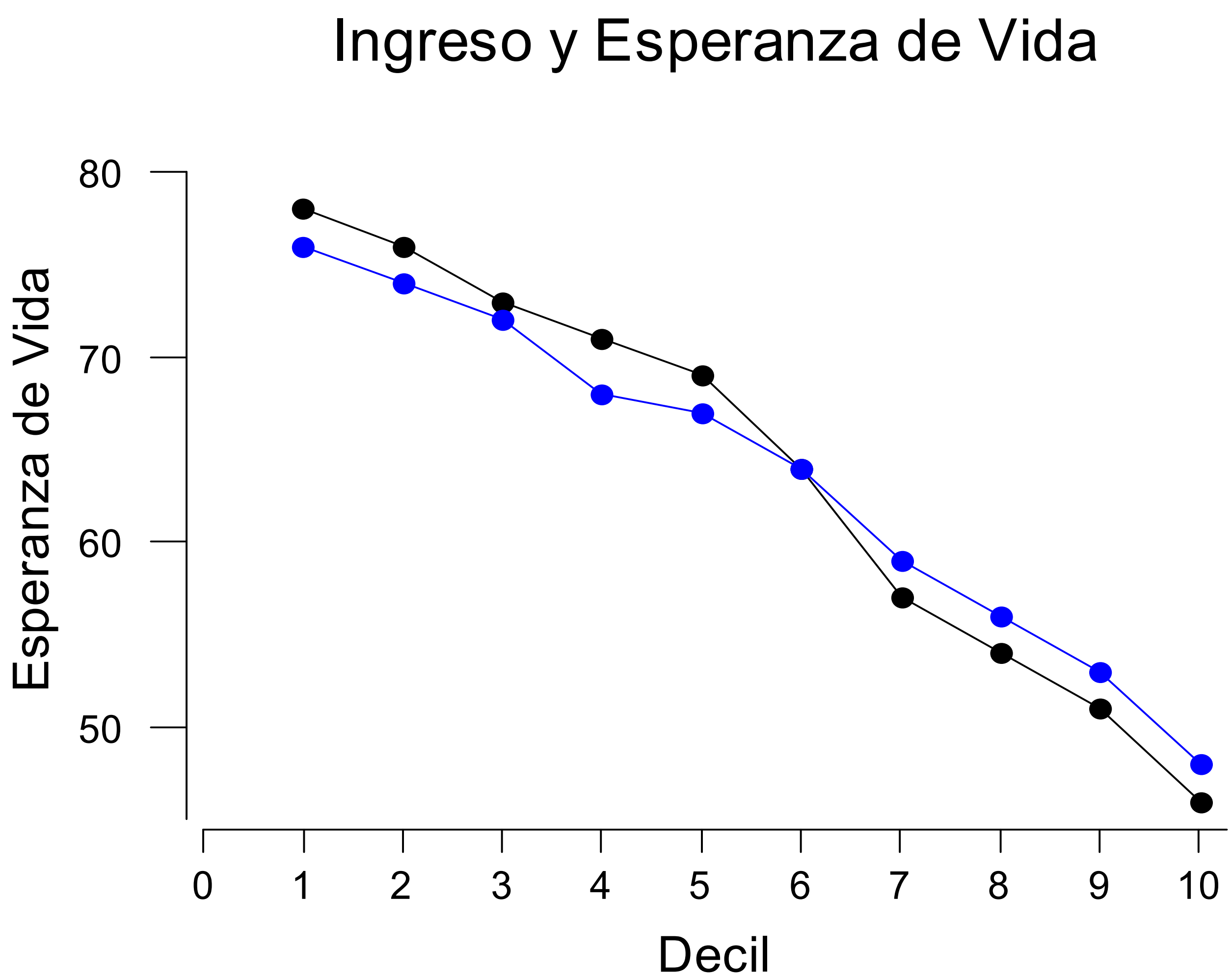
$$\begin{aligned} V^P(h,b) &= \int_I^{T(g)} e^{-\rho(a-I)} u(c(a))da \\ &+ e^{-\alpha_0+\alpha_1 f} \int_0^I e^{-\rho(a+B-I)} u(c_k(a))da \\ &+ e^{-\alpha_0+\alpha_1 f} e^{-\rho B} V^k(h_k(I), b_k) \end{aligned}$$

- Familias eligen:
  - Inversión en capital humano.
  - Inversión en capital de salud
- Modelo predice:
  - Escolaridad.
  - Fertilidad.
  - Esperanza de vida.

- Restricción presupuestaria:

$$
\begin{aligned}
& \int_I^{T(g)} e^{-r(a-I)} c(a)da + e^f \int_0^I e^{-r(a+B-I)} c_k(a)da \\
& + \int_I^R e^{-r(a-I)} x(a)da + \\
& e^f \int_6^I e^{-r(a+B-I)} x_k(a)da + e^f e^{-rB} b_k \\
& + e^f e^{-r(B+6-I)} x_E + e^f e^{-r(B-I)} g_k \\
\leq \; & \int_I^R e^{-r(a-I)} wh(a)(1-n(a))da \\
& + e^f \int_6^I e^{-r(a+B-I)} [wh_k(a)(1-n_k(a))]da + b.
\end{aligned}
$$


Ingreso, Fertilidad y Gasto en Salud
Fertilidad Gasto en Salud
Gasto Salud (% PBI)
Numero de Hijos
Decil
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10


Ingreso y Esperanza de Vida
Esperanza de Vida
80
70
60
50
0
1
2
3
4
5
6
7
8
9
10
Decil

- Teoría y evidencia son consistentes.

- Qué papel juega el capital humano?

Fertilidad: Sin Capital Humano			
Decil	Datos	Modelo	Sin Cap. Humano
90-100	1.74	2.22	2.11
40-50	3.37	3.71	2.23
0-10	5.66	5.82	2.56

Fluctuaciones y Crecimiento

- Evidencia: Modelo estadístico

$$\gamma_{it} = \beta X_{it} + \lambda \sigma_i + u_{it}$$

$$u_{it} = \sigma_i \epsilon_{it}, \qquad \epsilon_{it} \sim N(0,1).$$

- Resultados:
  - Correlación positiva: Kormendi y Meguire
  - Correlación negativa: Ramey y Ramey.
  - Correlación es fragil: Barlevy.

Table I: Growth and Volatility I

Variables	(1) (92−*Country*) N = 2,184	(2) (92−*Country*) N = 2,184	(3) (*OECD*) N = 888	(4) (*OECD*) N = 888
Constant	0.07 (3.72)	0.08 (3.73)	0.16 (5.73)	0.16 (4.48)
σ_i	−0.21 (−2.61)	−0.109 (−1.22)	−0.39 (−1.92)	−0.401 (−1.93)
Average I/Y	0.13 (7.63)	0.12 (6.99)	0.07 (2.76)	0.071 (2.67)
Average γ_{Pop}	−0.06 (−0.38)	−0.115 (−0.755)	0.21 (0.70)	0.230 (0.748)
H_0	0.0008 (1.18)	0.0007 (1.03)	0.0001 (2.00)	0.0001 (1.954)
Y_0	−0.009 (−3.61)	−0.009 −3.53)	−0.017 (−5.70)	−0.017 (−4.7445)
$\sigma_{\ln(I/Y)}$	-	−0.023 (1.81)		0.007 (0.22)

Note: t-statistics in parentheses

Source: Columns (1) and (3) Ramey and Ramey (1995)

Columns (2) and (4), Barlevy (2002)

Teoria I: Endogeneidad e Identificación

$$U = E\left[\int_0^\infty e^{-\rho t}\frac{c_t^{1-\theta}}{1-\theta}dt \,|F_0\right].$$

$$\begin{aligned} dk_t &= [Ak_t^\omega h_t^{1-\omega} - \delta_k k_t - x_t - c_t]dt + \sigma A k_t^\omega h_t^{1-\omega} dW_t, \\ dh_t &= -\delta_h h_t + x_t dt +, \end{aligned}$$

- Capital físico y humano.

- Equilibrio:

$$k_t = \alpha X_t, \; h_t = (1-\alpha)X_t.$$

- $\hat{F}(\alpha) \equiv A\alpha^\omega(1-\alpha)^{1-\omega}$.

- Naturaleza de la solución depende de la varianza "verdadera" (σ).

- Si $\sigma \leq \bar{\sigma}$ [caso A]: $\alpha^* = \omega$

$$\begin{aligned}\gamma_A &= \frac{\hat{F}(\omega) - (\rho + \delta)}{\theta} - \frac{1-\theta}{2}\sigma^2 \hat{F}(\omega)^2, \\ \sigma_{\gamma_A} &= \sigma \hat{F}(\omega).\end{aligned}$$

- Relación entre σ_γ y γ depende de la magnitud de θ.

- Si $\sigma \leq \bar{\sigma}$ [caso B]

$$\begin{aligned}
\gamma_B &= \frac{1}{\theta}[\frac{1+\theta}{2}\frac{1}{\theta\sigma^2} - (\rho+\delta)], \\
\sigma_{\gamma_B} &= \frac{1}{\theta\sigma}.
\end{aligned}$$

- Relación entre σ y σ_γ.

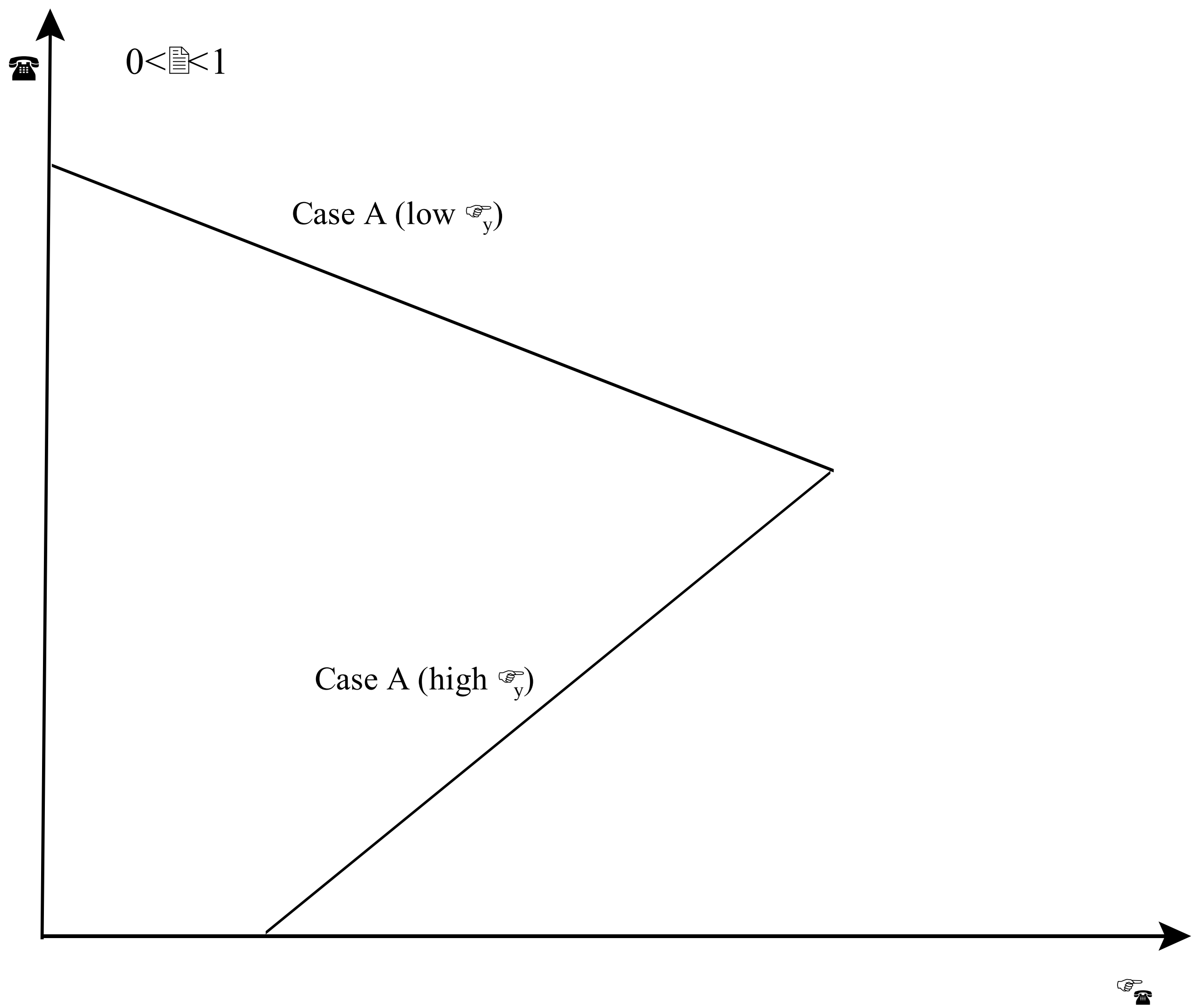


Figure 1: The mapping between σ_γ and γ. $[0 < \theta < 1]$

Teoría II: Origen de las Fluctuaciones

- Tecnología:

$$y_t = c_t = c_t = A^* \hat{z}_t n_t h_t,$$

$$dh_t = [1 - \delta + B(1 - n_t)]h_t dt + \sigma_h [1 - \delta + B(1 - n_t)]h_t dW_t,$$

$$d\hat{z}_t = \frac{\alpha}{(1-\alpha)^2} \frac{\sigma_w^2 + \sigma_m^2}{2} \hat{z}_t dt + \frac{\alpha}{(1-\alpha)} \hat{z}_t (\sigma_w dW_t + \sigma_m dM_t).$$

- Resultados ($\sigma_w = 0$):

$$x = 1 - \delta + B(1 - n)$$

$$\gamma = x + \frac{\alpha}{(1-\alpha)^2} \frac{\sigma_m^2}{2}$$

$$\sigma_\gamma^2 = (\sigma_h x)^2 + (\frac{\alpha}{1-\alpha} \sigma_m)^2$$

- Aumentos de σ_h generan correlación negativa entre γ y σ_γ.
- Aumentos de σ_m generan correlación positiva entre γ y σ_γ.
- Identificación?

Teoría III: Shocks Fiscales

- Retornos privados:

$$\begin{aligned} dk_t &= (r_k k_t - c_{1t})dt + \sigma_k k_t dW_t, \\ db_t &= (r_b b_t - c_{2t})dt + \sigma_b b_t dW_t, \\ c_t &= c_{1t} + c_{2t}, \end{aligned}$$

- $k_t =$ capital

- $b_t=$ deuda del gobierno.

- Tecnología:

$$\begin{aligned} dk_t &= (Ak_t - c_t)dt + \sigma A k_t dW_t - dG_t, \\ dG_t &= gAk_t dt + g'\sigma A k_t dW_t. \end{aligned}$$

- Gobierno:

$$dT_t = \tau A k_t dt + \tau' \sigma A k_t dW_t,$$

$$B_t + dG_t - dT_t = p_t dB_t,$$

- Equilibrio:

$$\begin{aligned} r_k &= (1-\tau)A, \\ \sigma_k &= (1-\tau')\sigma A. \end{aligned}$$

$$\gamma = \frac{(1-\tau)A-\rho}{\theta} - \left(\frac{1-\theta-\tau'+\theta g'}{1-g'}\right)\sigma_\gamma^2 \qquad (2a)$$

$$\sigma_\gamma = \sigma(1-g')A. \qquad (2b)$$

- Signo de la relación entre σ_γ y γ depende de parámetros y de la política fiscal

Reflexiones Finales

- Capital humano:
  * Amplifica cambios en productividad.
  * Distorsiones tienen efectos significativos.
  * Identificación y dinámica
- Fluctuaciones y crecimiento:
  * Teoría da resultados ambiguos.
  * Evidencia empírica es difícil de interpretar.
  * Próxima generación: detalles micro.

Additional Material:

Fertility: U.S. vs Europe

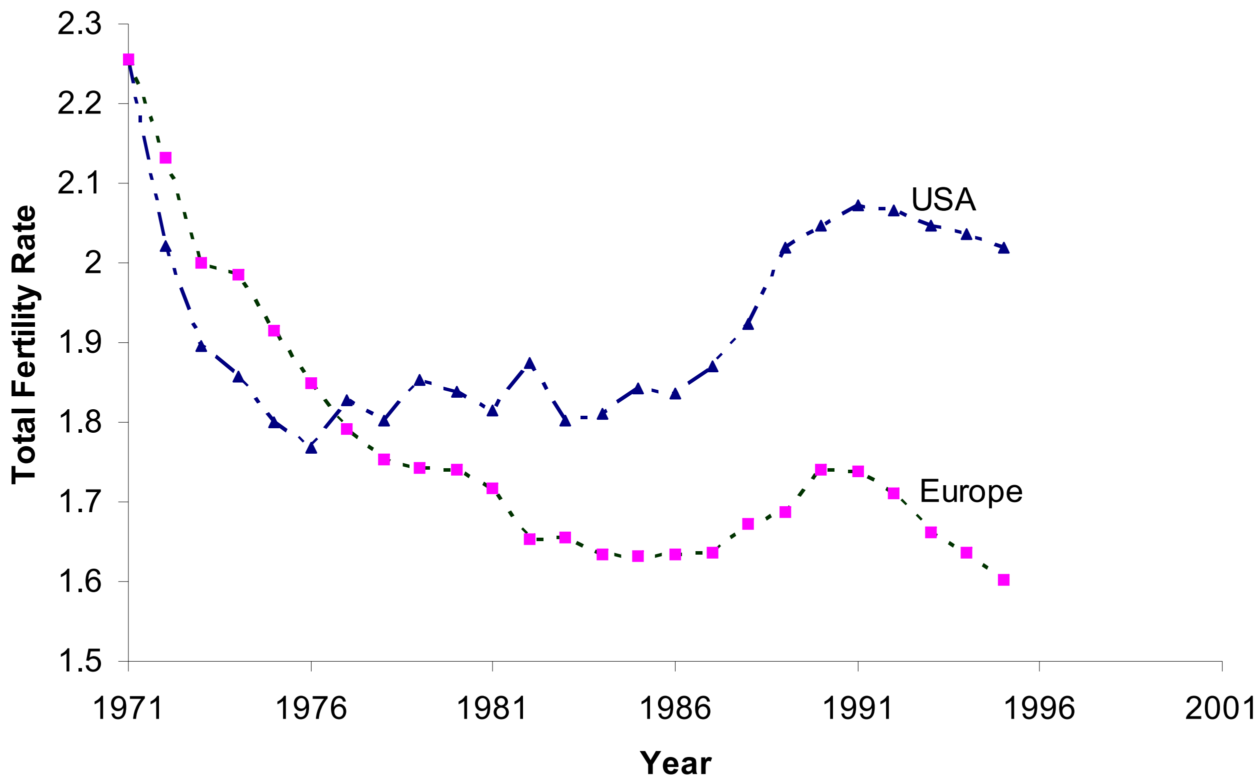


Figure 1: Diferencias de Fertilidad Europe-USA, 1970-1995

Regímenes Jubilatorios e Impuestos

– Qué pasó en Europa?

– Una interpretación: el aumento de los beneficios jubilatorios bajo la "demanda" de hijos ya que se creó otro "activo" que da retornos durante la vejez. [Boldrin y Jones]

– Cómo se financió la expansión del sistema?

– Fuertes aumentos de impuestos!

– Mecanismo:

  * Impuestos afectan el retorno al capital humano.
  * Reducen la ganancia (para los hijos) de acumular capital humano.
  * Reduce su utilidad.
  * Reduce la "demanda" de hijos por parte de los padres.

* Fertilidad cae.

Usando el Modelo: Lejano Oriente vs. América Latina

– Son los países de América Latina y el Lejano Oriente distintos?

– Posibilidad: Esta teoría no se aplica estos dos grupos (milagros y desastres).

– Ejercicio: "Simular" la experiencia de estas dos regiones desde la década de los 60.

Milagros: Lejano Oriente

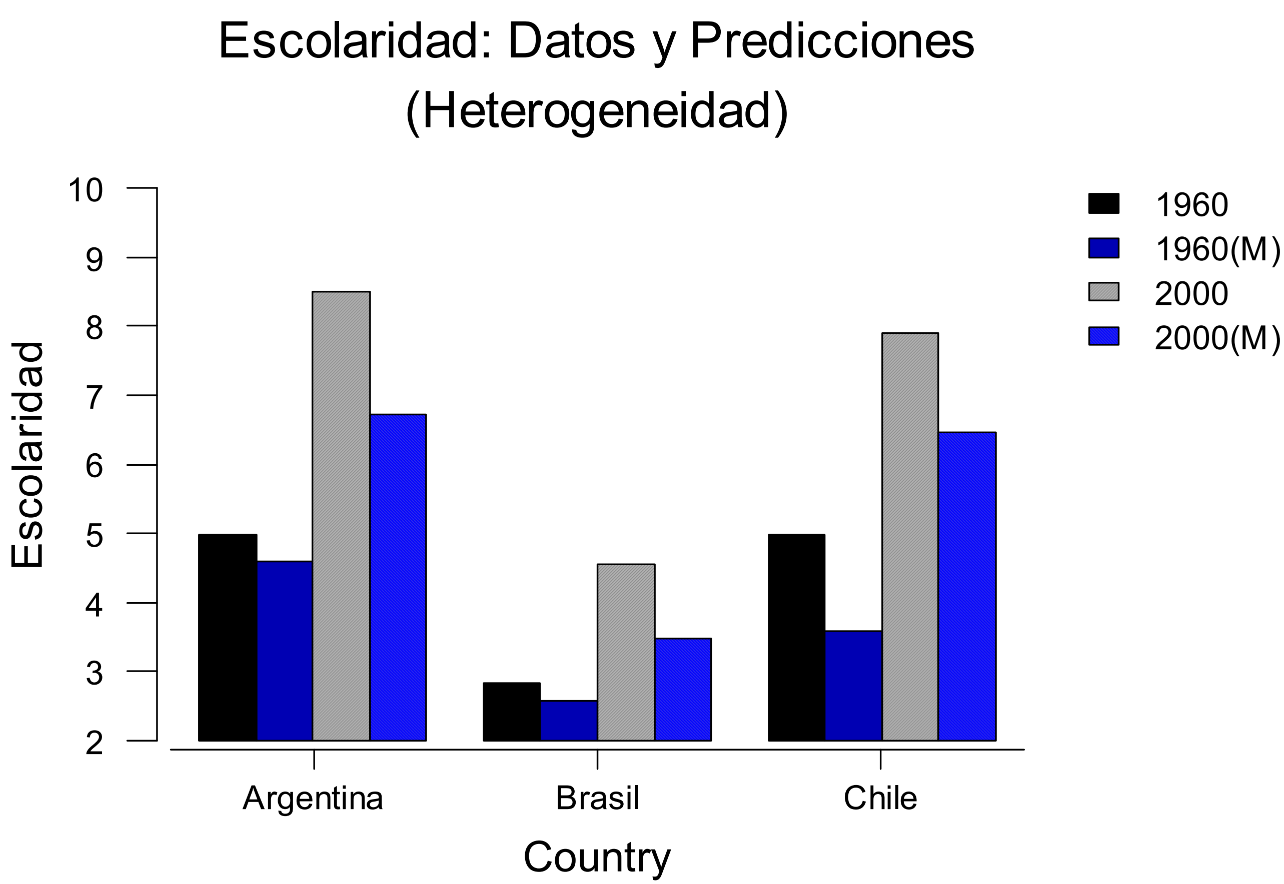
– El modelo es bastante exitoso reproduciendo la experiencia de:

  * Corea,
  * Hong Kong
  * Malasia,
  * Singapur
  * Taiwan.

– El modelo predice (aproximadamente) los cambios en:
  * Niveles de escolaridad.
  * La relacion inversión-producto.
– Factores determinantes:
  * Aumentos en la productividad.
  * Cambio demográfico.

Desastres: América Latina

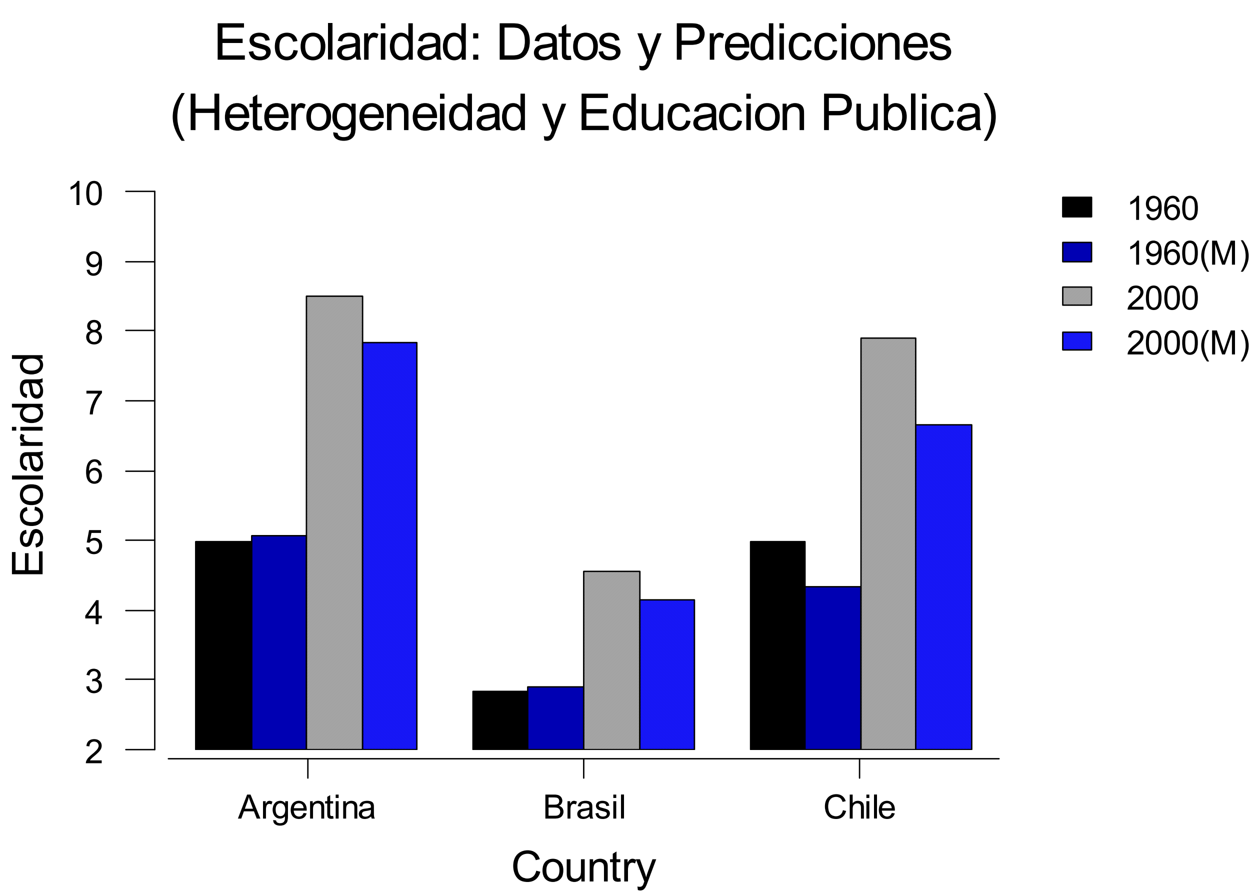
- Como en el caso anterior, los factores que varían son:
  - Estructura demográfica.
  - Productividad.
- Heterogeneidad: diferencias en abilidad natural.
  - Medición.


Escolaridad: Datos y Predicciones
(Heterogeneidad)
Escolaridad
10
9
8
7
6
5
4
3
2
Argentina
Brasil
Chile
Country
1960
1960(M)
2000
2000(M)

- "Misterio de la educación"
- 1960: Datos y Modelo coinciden
- 2000: El nivel de escolaridad excede las prediciones del model
  - Promedio para América Latina: 69%
  - Excluyendo Perú y Venezuela: 29%
  - Argentina, Brasil y Chile: 26%
- Excesivos gastos en educación.
- Conjetura: Calidad de (y acceso a) la educación es subóptimo.

Educación Pública

- Todos los estudiantes reciben los mismos recursos educativos.
- Tienen distintos niveles de abilidad.
- Supuestos:
  - Gasto Secundaria/Gasto Primaria en 1960 y 2000
  - Cambios en productividad y estructura poblacional.


Escolaridad: Datos y Predicciones
(Heterogeneidad y Educacion Publica)
Escolaridad
10
9
8
7
6
5
4
3
2
Argentina
Brasil
Chile
Country
1960
1960(M)
2000
2000(M)

- Resultado:
  - Modelando un sistema de educacion pública reduce la “paradoja educativa” del 26% al 12%.
- Importa la asignación de recursos entre niveles?
- Experimento: Reasignación de los recursos del nivel terciario al nivel primario.

- Manteniendo la participación del gasto educativo el el producto.
  - Efecto de largo plazo en el producto:
    - Argentina = 12%
    - Brasil = 19%
    - Chile = 17%.
- Dado que este es un aumento permanente, no es despreciable!

La Experiencia de Otros Países

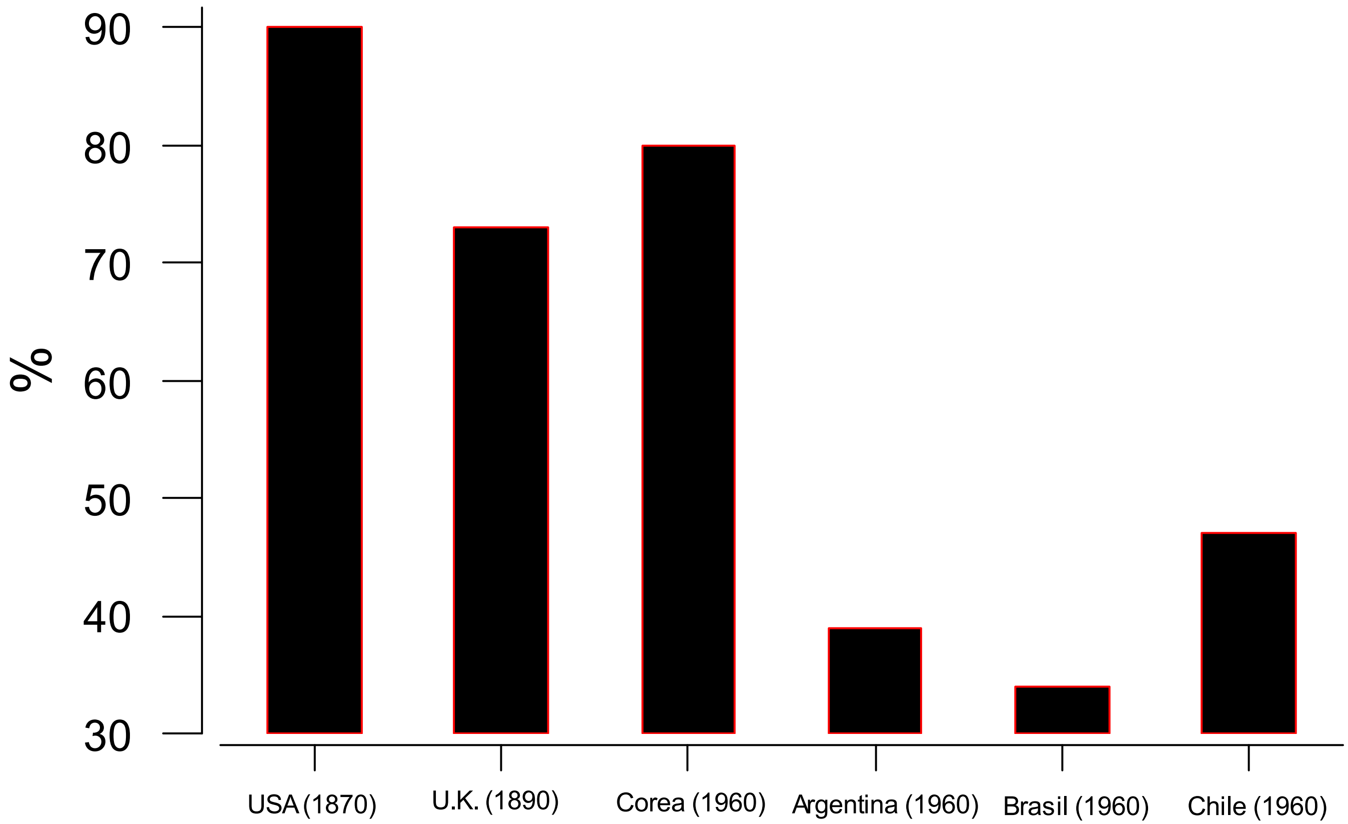
– Cómo asignan otros países los gastos educacionales?

– Dos tipos de ejemplos:

  * Países desarrollados cuando su nivel de escolaridad es comparable a los niveles de ABC.

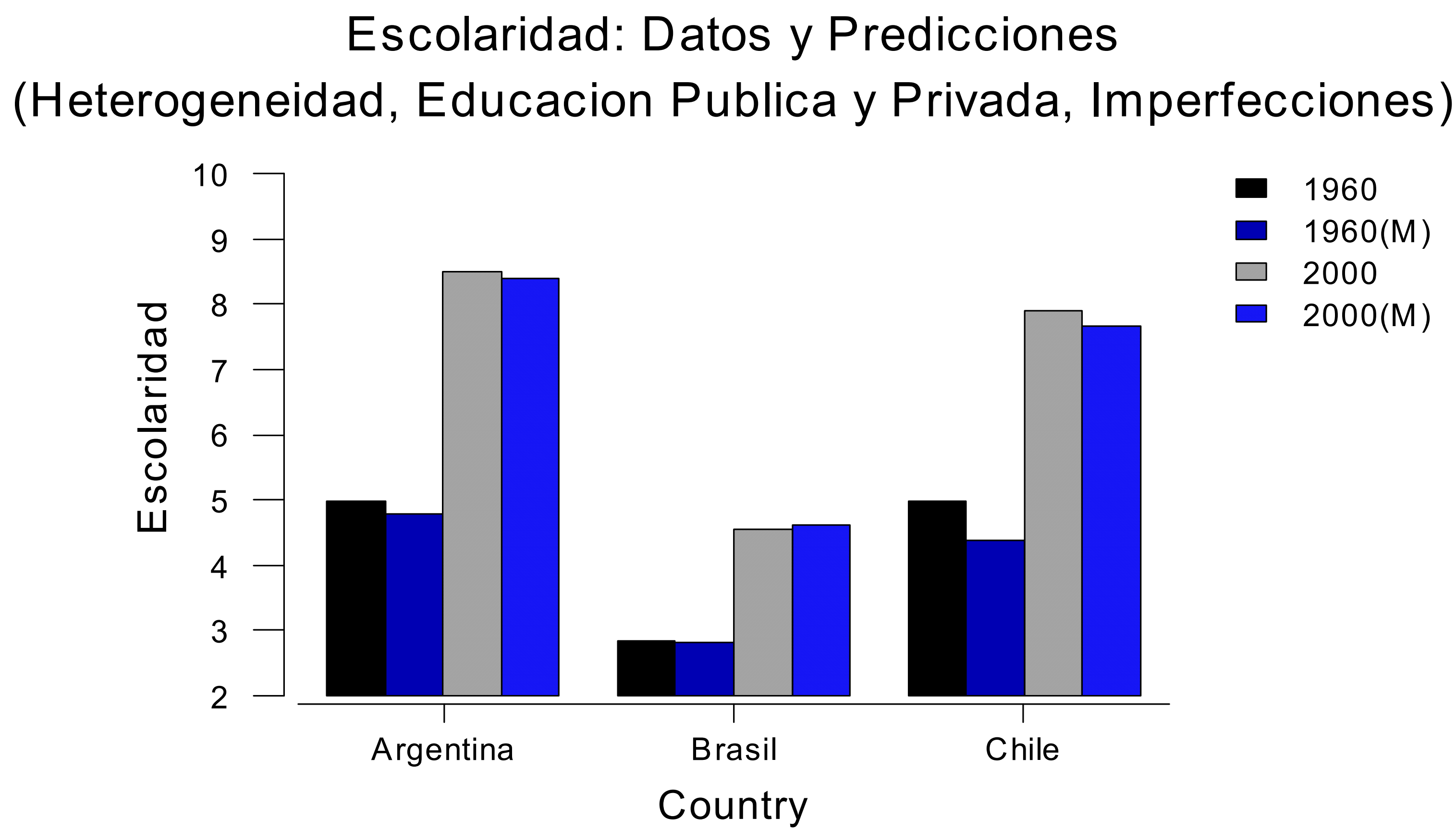
  * Países que crecieron rápido (economías milagro)

Gasto en Escolaridad Primaria
(Porcentaje del Total)



Educación Pública y Privada: Mercados de Capital Imperfectos

- Educación Privada: individuos compran recursos educativos.
- El sistema financiero no da préstamos educativos.
- Los padres sí.


Escolaridad: Datos y Predicciones
(Heterogeneidad, Educacion Publica y Privada, Imperfecciones)
Escolaridad
10
9
8
7
6
5
4
3
2
1960
1960(M)
2000
2000(M)
Argentina
Brasil
Chile
Country

- El “misterio de la educación” desaparece:
  * La mitad del exceso educativo se debe a mala asignación de recursos.
  * La otra mitad a imperfecciones en el mercado de capitales.

Fluctuaciones y Crecimiento

Table II: Growth and Volatility II

Independent Variable	92-Country Sample (2,208 observations)	OECD Sample (888 observations)
Constant	0.00132 (0.022)	0.095 (1.89)
Within-phase volatility (σ_{iw})	2.63 (4.69)	0.90 (1.44)
Between-phase volatility (σ_{ib})	−2.65 (−6.35)	−1.11 (−2.33)
Average investment share of GDP	−0.01 (−0.26)	−0.004 (−0.073)
Average population growth rate	0.58 (1.24)	0.28 (0.62)
Initial human capital	0.001 (0.66)	−0.00001 (−0.096)
Initial per capita GDP	0.002 (0.25)	−0.0008 (−1.30)

Note: t-statistics in parentheses.

Source: Kroft and Lloyd-Ellis (2002).

Table III: Growth and Volatility III

Independent Variable	(1)	(2)	(3)
Volatility (σ_i)	−2.772 (0.282)	−1.700 (0.645)	−0.270 (0.091)
GDP per capita (1960)	−2.229 (0.235)	−1.856 (0.422)	−0.953 (0.220)
Human capital (1960)	0.037 (0.015)	0.040 (0.018)	0.026 (0.017)
Average investment share of GDP	0.083 (0.013)	0.143 (0.021)	0.120 (0.024)
Average population growth rate	−0.624 (0.153)	−0.562 (0.205)	−0.465 (0.465)
Volatility * GDP	0.340 (0.036)	—	-
Volatility * GDP (1960)	—	0.212 (0.082)	-
Volatility * M3/Y	-	-	0.004 (0.001)
R^2	0.77	0.58	0.57

Note: Sample 1950-1998. Robust standard errors in parentheses
Source: Fatás (2001

Distortions and TFP

– Technology

$$y_{it} = Ak_{it}^{\alpha} n_{it}^{\theta} a_i^{1-\alpha-\theta} \qquad 0 < (\alpha, \theta) < 1, \qquad \alpha + \theta < 1,$$

- a_i is interpreted as managerial ability

- $\theta_i = 1$ for all i.

Distortionary Policy

- Sector specific taxes and/or subsidies

- If market price is p, a producer in sector i faces a price equal to $p/(1-\tau_i^j)$,

- Solution

$$\begin{aligned} k_{it} &= (1-\tau_{it}^n)^{\frac{\theta}{1-\alpha-\theta}}(1-\tau_{it}^k)^{\frac{1-\theta}{1-\alpha-\theta}}C_k \\ n_{it} &= (1-\tau_{it}^n)^{\frac{1-\alpha}{1-\alpha-\theta}}(1-\tau_{it}^k)^{\frac{\alpha}{1-\alpha-\theta}}C_n \end{aligned}$$

  where C_k and C_n are constants independent of i.

- Equilibrium: economy-wide values denoted by k_t and n_t, respectively.

- View a particular pair $(1-\tau^n_{it}, 1-\tau^k_{it})$ as being drawn from some joint distribution.

$$
\begin{aligned}
k_{it} &= k_t(1-\tau^n_{it})^{\frac{\theta}{1-\alpha-\theta}}(1-\tau^k_{it})^{\frac{1-\theta}{1-\alpha-\theta}}E[(1-\tau^n_{it})^{\frac{\theta}{1-\alpha-\theta}}(1-\tau^k_{it})^{\frac{1-\theta}{1-\alpha-\theta}}]^{-1} \\
n_{it} &= n_t(1-\tau^n_{it})^{\frac{1-\alpha}{1-\alpha-\theta}}(1-\tau^k_{it})^{\frac{\alpha}{1-\alpha-\theta}}E[(1-\tau^n_{it})^{\frac{1-\alpha}{1-\alpha-\theta}}(1-\tau^k_{it})^{\frac{\alpha}{1-\alpha-\theta}}]^{-1}
\end{aligned}
$$

- Aggregate output

$$
y_t = \Delta(\sigma_n, \sigma_k, \rho) A k_t^\alpha n_t^\theta
$$

where

$$
\begin{aligned}
\Delta(\sigma_n, \sigma_k, \rho) &= \exp\{-\frac{1}{(1-\alpha-\theta)}[\frac{\theta(1-\alpha)\sigma_n^2}{2} \\
&\quad + \frac{\alpha(1-\theta)\sigma_k^2}{2} + \rho\alpha\theta\sigma_n\sigma_k]\}
\end{aligned}
$$

- If either σ_n or σ_k are positive, $\Delta(\sigma_n, \sigma_k, \rho) < 1$, and *actual* TFP falls short *potential* TFP.

- The μ_j do not enter in $\Delta(\sigma_n, \sigma_k, \rho)$

- $\Delta(\sigma_n, \sigma_k, \rho)$ is strictly convex in $(\sigma_n, \sigma_k, \rho)$.

- Example: ($\alpha = \theta = 0.4$), and $\sigma_n = \sigma_k = \sigma$.

- The values of (σ_n, σ_k) should be interpreted as measures of the cross-sectional variability of incentives relative to the mean level of distortion. Thus, a value of 0.5 corresponds to the case in which the coefficient of variation is 50%. I considered values of σ in the interval [0.1,0.7] with increment size equal to 0.1, and several values of the correlation coefficient ρ. The results are in Table 1

Table 1: TFP Gap

σ	ρ					
	-0.8	-0,4	0.0	0.4	0.8	1.0
0.1	.99	.99	.98	.98	.98	.98
0.2	.98	.97	.95	.94	.93	.92
0.3	.95	.92	.90	.87	.85	.83
0.4	.91	.87	.83	.78	.74	.73
0.5	.87	.80	.75	.68	.63	.61
0.6	.82	.73	.65	.58	.52	.49
0.7	.76	.65	.56	.47	.41	.38

- Let κ_{it} be the capital-labor ratio in sector i at time t.

$$\kappa_{it} = C\frac{k_t}{n_t}\frac{1-\tau_{it}^k}{1-\tau_{it}^n}$$

where C is a constant.

- Let the variance of the log of κ_{it} be denoted $\sigma^2(\ln \kappa_{it})$. Then it follows that,

$$\sigma^2(\ln \kappa_{it}) = \sigma_k^2 + \sigma_n^2 - 2\rho\sigma_k\sigma_n.$$

Thus, if the is no cross-sectional variability in tax/subsidies, $\sigma^2(\ln \kappa_{it})$ should be zero. Evidence of variability, and especially changes over time, is indirect evidence for the presence of distortions.

- Another variable that captures the relevant features of the tax code is the unit price of the sector-specific resource, a_i. The variance of $\ln p_{it}$ is given by,

$$\begin{aligned} \sigma^2(\ln p_{it}) &= \left(\frac{\alpha}{1-\alpha-\theta}\right)^2 \sigma_k^2 + \\ &\quad \left(\frac{\theta}{1-\alpha-\theta}\right)^2 \sigma_n^2 - \frac{2\alpha\theta}{(1-\alpha-\theta)^2}\rho\sigma_k\sigma_n. \end{aligned}$$

Human Capital Wealth of Nations

Household's decision problem

$$V^P(h,b) = \max \int_I^T e^{-\rho(a-I)} u(c(a))da + e^{-\alpha_0+\alpha_1 f} e^{-\rho B} V^k(h_k(B+I), b_k)$$

subject to

$$\int_I^T e^{-r(a-I)} c(a)da + \int_I^R e^{-r(a-I)} x(a)da + e^f \int_B^{B+I} e^{-r(a-I)} \left[c_k(a) + x_k(a)\right] da$$
$$+ e^f e^{-rB} b_k + e^f e^{-r(B+6-I)} x_E$$
$$\leq \int_I^R e^{-r(a-I)} wh(a)(1-n(a))da +$$
$$e^f \int_B^{B+I} e^{-r(a-I)} wh_k(a)(1-n_k(a))da + b,$$

Human Capital Production

- Human capital production technology

$$\dot{h}(a) = z_h[n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [I, R),$$

- Production technology for early childhood human capital

$$h_k(B+6) = h_B x_E^{\upsilon}, \qquad h_B \qquad given.$$

Equivalent to Income Maximization

- If $r = \rho + [\alpha_0 + (1 - \alpha_1) f]/B$ (steady state), equilibrium allocation solves

$$\max \int_6^R e^{-r(a-6)} [wh(a)(1 - n(a)) - x(a)] da - x_E$$

subject to

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [6, R),$$

and

$$h(6) = h_E = h_B x_E^{\upsilon}$$

with h_B given.

- Here a indicates an individual's age.

Equilibrium Schooling Level (s)

- Define schooling s as the time period for which $n = 1$.

- Optimal s satisfies

$$\Psi(s; r, R) = \frac{z_h^{1-\upsilon} w^{\gamma_2 - \upsilon(1-\gamma_1)}}{h_B^{1-\gamma}}.$$

where (at the solution) $\Psi'(s; r, R) > 0$.

Determinants of Schooling

- Macro effects
  - If $\gamma_2 = \upsilon = 0$, w (TFP) has no impact on s
  - If $\gamma_2 - \upsilon(1 - \gamma_1) > 0$, higher w (TFP) induces higher s
- Micro effects
  - Higher h_B implies lower s
  - Higher z_h implies higher s

Investments over the Life-Cycle

- During the period of schooling, level of x increases with age

$$x(a) = \left(\frac{\gamma_2 w}{r+\delta_h}\right) C_h(z_h, w, r) m(6+s)^{\frac{1}{1-\gamma}} e^{\frac{r+\delta_h(1-\gamma_1)}{(1-\gamma_2)}(a-s-6)}, \;\; a \in [6, 6+s),$$

- During the period of training, x (and n) decreases with age

$$x(a) = \left(\frac{\gamma_2 w}{r+\delta_h}\right) C_h(z_h, w, r) m(a)^{\frac{1}{1-\gamma}}, \qquad a \in [6+s, R).$$

where

$$m(a) = 1 - e^{-(r+\delta_h)(R-a)},$$

Development and Schooling Quality

- Consider two economies with $w' > w$
- Consider two individuals in these countries who possess the same level of schooling.
- They need to differ along some dimension, say h_B.
- Then, the elasticity of $h(s+6)$ with respect to w is $\gamma_2/(1-\gamma)$

Equilibrium Age-Earnings Profiles

- Need to take a stand on who pays for training and how much.

- Fraction π of post-schooling expenses in market goods are paid for by employers.

$$y(a) = wh(a)(1 - n(a)) - \pi x(a).$$

$$\begin{aligned} y(a) \;&=\; \left[\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r+\delta_h)^{\gamma}}\right]^{\frac{1}{1-\gamma}} w\{\gamma_1 e^{-\delta_h(a-6-s)} \frac{m(6+s)^{\frac{1}{1-\gamma}}}{r+\delta_h} \\ &-(\gamma_1 + \pi\gamma_2)\frac{m(a)^{\frac{1}{1-\gamma}}}{r+\delta_h} + \frac{e^{-\delta_h(a-R)}}{\delta_h}\int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} [(1 - x^{\frac{r+\delta_h}{\delta_h}}]^{\frac{\gamma}{1-\gamma}} dx\}. \end{aligned}$$

- Tempting to view this as a foundation for Mincer-style regressions.

- Issues.

TFP and the Stock of (end of school) Human Capital

- End of schooling human capital is given by

$$h(6+s) \propto w^{\frac{\gamma_2}{1-\gamma}}\left(\frac{1}{r+\delta}\right)^{\frac{1}{1-\gamma}}.$$

- Suppose that r is constant across countries, then

$$h(6+s) \propto z^{\frac{\gamma_2}{(1-\theta)(1-\gamma)}}.$$

- Let $\gamma_1 = 0.6, \gamma_2 = 0.33$ and $\theta = 0.315$. Then $\frac{\gamma_2}{(1-\theta)(1-\gamma)} = 6.88$

Quality vs. Quantity of Human Capital

- Model implies $\frac{h(6+s)|_{US}}{h(6+s)|_P} = 8.7$

- Decompose to differences in quantity and quality. Let

$$h(6+s) = q_i e^{\phi_i s}$$

- Assume that $\phi_{US} = \phi_P = 0.10$

- Then

$$\begin{aligned} \frac{q_{US}}{q_P} &= \left(\frac{z_{US}}{z_P}\right)^{\frac{\gamma_2}{(1-\theta)(1-\gamma)}} e^{\phi_P s_P - \phi_R s_R} \\ &= 3.2 \end{aligned}$$

- Ratio of quality of human capital is a factor of 3.2

Aggregation and Equilibrium

- Population: $N(a,t) = e^{f} N(B, t-a) \rightarrow \phi(a) = \eta \frac{e^{-\eta a}}{1-e^{-\eta T}}$. [$\eta = f/B =$ Pop growth]

- $\bar{h} = \int_{6+s}^{R} h(a)(1-n(a))\phi(a)da$

- $r = \rho + \frac{\alpha_0}{B} + (1-\alpha_1)\eta$

- $p_k(r+\delta_k) = zF_k(\kappa, 1)$, [$\kappa = k/\bar{h}$]

- $w = zF_h(\kappa, 1)$

Calibration

- Functional forms

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

and

$$F(k, h) = zk^{\theta}h^{1-\theta}.$$

- Irrelevant Parameters
  - z, p_k, h_I
  - σ, υ_0 and υ_1

- Based on prior work
  - $\rho = 0.04$ Cooley and Prescott, 1995
  - $\delta_k = 0.06$ Cooley and Prescott, 1995
  - $\pi = 0.5$ (goods component of training not reflected in wages, sensitivity)
  - $\alpha_1 = 1$ ($p_k K/Y$ constant across countries - Hsieh and Klenow, 2003)
    * also implies r constant across countries

- Left with 7 parameters $\theta, \alpha_0, \delta_h, z_h, \gamma_1, \gamma_2$ and ν.

1. Capital's share of income of 0.33. Source: NIPA
2. Capital Output Ratio of 2.54. Source: NIPA
3. Earnings at age 55/Earnings at age R of 0.8. Source: SSA
4. Earnings at age 50/Earnings at age 25 of 2.17. Source: SSA
5. Years of schooling of 12. Source: Barro and Lee
6. Schooling expenditures per pupil/GDP per capita of 0.21. Source: OECD
7. Pre-primary expenditures per pupil/GDP per capita of 0.14. Source: OECD

Parameter	θ	α_0	δ	z_h	γ_1	γ_2	ν
Value	0.315	0.75	0.018	0.361	0.63	0.3	0.55

- What happens with a lower γ (between 0.7 and 0.9)?
  - Say we adjust z_h upward to keep schooling constant
  - Then age earnings profile too steep.

- What about when γ is even lower?
  - After all, when $\gamma = 0$, the age earnings profile is flat!
  - With γ below 0.6 or so, not possible to match s.

East Asia vs. Latin America: Theory

Public Education

- Human capital accumulation technology during schooling

$$\dot{h}(a) = z_h h(a)^{\gamma_1} x_G^{\gamma_2}, \qquad a \in [6, s+6), \qquad h(6) = h_E.$$

- Eliminated depreciation (small role in early years)

- x_G is allowed by schooling level.

- If financial markets “work well” the education problem is

$$
\begin{aligned}
W(h_B, s) &= \max[-(1+\tau^e)x_e + \\
&\quad e^{-r(6+s)} \int_{s+6}^{R} e^{-r(a-6-s)}[wh(a)(1-n(a)) - x(a)]da], \\
&= \max_{x_e, s}[-(1+\tau^e)x_e + e^{-r(6+s)} V(h(6+s), s)],
\end{aligned}
$$

- $V(h(6+s), s)$ = present discounted value of net labor income in the post-schooling period.

- Early childhood human capital

$$
\begin{aligned}
\dot{h}(a) &= z_h h(a)^{\gamma_1} x_G^{\gamma_2}, \qquad a \in [6, s+6), \qquad h(6) = h_{E,} \\
h_E &= h(6) = z_e x_e^{\upsilon}.
\end{aligned}
$$

- Human capital level at age a, $a \in [6, s+6)$

$$
h(a) = [h(6)^{1-\gamma_1} + (1-\gamma_1) z_h x_G^{\gamma_2}(a-6)]^{\frac{1}{1-\gamma_1}}
$$

- Optimality

$$
\begin{aligned}
q(6+s)z_h h(6+s)^{\gamma_1} x_G^{\gamma_2} &= wh(6+s) - \frac{w\gamma}{\gamma_1} q(6+s)^{\frac{1}{1-\gamma}} [\frac{z_h}{w^{1-\gamma_2}} (\frac{\gamma_2}{\gamma_1})^{\gamma_2}]^{\frac{1}{1-\gamma}} \\
&\quad -q(6+s)\delta h(6+s) + \\
&\quad \left(\frac{r+\delta}{w}\right)^{\frac{\gamma}{1-\gamma}} [\frac{\gamma_2^{\gamma_2} \gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r+\delta)^{\gamma}}]^{\frac{1}{1-\gamma}} q(6+s)^{\frac{1}{1-\gamma}},
\end{aligned}
$$

- and optimal choice of early childhood

$$\frac{De^{rs}}{\left[\left(D^{\upsilon}\left(\frac{\upsilon}{1+\tau^{e}}\right)^{\upsilon} z_e\right)^{1-\gamma_1} + (1-\gamma_1) z_h x_G^{\gamma_2} s\right]^{\frac{1}{1-\gamma_1}}} = \frac{w}{r+\delta} m(6+s) \tag{5}$$

and

$$\begin{aligned} D \;=\;& z_h x_G^{\gamma_2}\left(\frac{w}{r+\delta}\right)^{1-\gamma_1}\left(\frac{m(6+s)}{e^{rs}}\right)^{1-\gamma_1}\frac{m(6+s)}{r+\delta-\delta m(6)} D^{\gamma_1} \qquad (6)\\ &+\left[\frac{w^{1-\gamma_1}\gamma_2^{\gamma_2}\gamma_1^{\gamma_1} z_h}{r+\delta}\right]^{\frac{1}{1-\gamma}}\left(\gamma\gamma_1^{-\frac{1}{1-\gamma}} - 1\right)\frac{m(6+s)}{e^{rs}}\frac{m(6+s)^{\frac{1}{1-\gamma}}}{r+\delta-\delta m(6+s)} \end{aligned}$$

- Multiple schooling levels

$$h(6+s) = [h(6)^{1-\gamma_1} + (1-\gamma_1) z_h \bar{x}_E(s)]^{\frac{1}{1-\gamma_1}},$$

$$\bar{x}_E(s) \begin{cases} x_P^{\gamma_2} s & 0 \leq s \leq s_P \\ x_P^{\gamma_2} s_P + x_S^{\gamma_2}(s - s_P) & s_P \leq s \leq s_P + s_S \\ x_P^{\gamma_2} s_P + x_S^{\gamma_2} s_P + x_T^{\gamma_2}(s - s_S - s_P) & s_P + s_S \leq s \end{cases}.$$

Fertility: Some Theoretical Results

Data

Table 1: World Distribution						
Decile	y (relative to US)	s	x_s	T	f (TFR/2)	p_k
90-100	0.921	10.93	3.8	78	0.85	1.02
80-90	0.852	9.94	4.0	76	0.9	1.11
70-80	0.756	9.72	4.3	73	1.0	1.06
60-70	0.660	8.70	3.8	71	1.2	1.04
50-60	0.537	8.12	3.1	69	1.35	1.52
40-50	0.437	7.54	2.9	64	1.6	1.77
30-40	0.354	5.88	3.1	57	2.05	1.56
20-30	0.244	5.18	2.7	54	2.5	1.93
10-20	0.146	4.64	2.5	51	2.7	2.11
0-10	0.052	2.45	2.8	46	3.1	2.78

Main Results

Table 2: Output and Schooling - Data and Model (Constant p_k)						
Decile	y (relative to US)	TFP	s		x_s	
			Data	Model	Data	Model
90-100	0.921	0.99	10.93	11.64	3.8	3.7
80-90	0.852	0.98	9.94	10.92	4.0	3.5
70-80	0.756	0.97	9.72	10.40	4.3	3.4
60-70	0.660	0.95	8.70	9.64	3.8	3.9
50-60	0.537	0.93	8.12	8.90	3.1	4.9
40-50	0.437	0.90	7.54	6.79	2.9	4.7
30-40	0.354	0.88	5.88	5.69	3.1	3.9
20-30	0.244	0.85	5.18	4.29	2.7	3.2
10-20	0.146	0.82	4.64	3.01	2.5	2.7
0-10	0.052	0.73	2.45	2.19	2.8	2.0

Changes in Relative Price of Capital

Table 3: Output and Schooling - Data and Model				
(Varying p_k)				
Decile	y	p_k	TFP	TFP
	(relative to US)		baseline	p_k varies
90-100	0.921	1.02	0.99	1.00
80-90	0.852	1.11	0.98	1.01
70-80	0.756	1.06	0.97	0.99
60-70	0.660	1.04	0.95	0.96
50-60	0.537	1.52	0.93	1.05
40-50	0.437	1.77	0.90	1.07
30-40	0.354	1.56	0.88	1.01
20-30	0.244	1.93	0.85	1.05
10-20	0.146	2.11	0.82	1.04
0-10	0.052	2.78	0.73	1.01

Demographics (as in U.S.)

Table 4: Output and Schooling - Data and Model					
Changing Demographics					
Decile	y		s		
	baseline	demog	Data	baseline	demog
90-100	0.921	0.913	10.93	11.64	11.70
80-90	0.852	0.851	9.94	10.92	11.21
70-80	0.756	0.756	9.72	9.40	10.2
60-70	0.660	0.664	8.70	8.64	9.33
50-60	0.537	0.572	8.12	7.30	8.56
40-50	0.437	0.483	7.54	6.49	7.92
30-40	0.354	0.402	5.88	5.49	7.12
20-30	0.244	0.331	5.18	4.29	5.97
10-20	0.146	0.251	4.64	3.01	4.79
0-10	0.052	0.123	2.45	2.19	4.04

Decomposition of Human Capital Differences

Table 5: Human Capital			
(relative to U.S.)			
Decile	y	h_B	$\bar{h}$
	(relative to US)	(relative to US)	(relative to US)
90-100	0.921	0.96	0.95
80-90	0.852	0.91	0.88
70-80	0.756	0.88	0.79
60-70	0.660	0.86	0.71
50-60	0.537	0.79	0.60
40-50	0.437	0.72	0.50
30-40	0.354	0.65	0.43
20-30	0.244	0.60	0.32
10-20	0.146	0.53	0.20
0-10	0.052	0.47	0.08

East Asia vs. Latin America

Fast Growers: East Asia

Table 1: East Asia

	$\Delta(Y/L)$	Years of Schooling			
	Data	Data		Model	
Country		1960	2000	1960	2000
Sing.	6.6	3.14	8.12	3.21	8.66
H.K.	9.09	4.74	9.47	4.36	8.87
Mal.	4.49	2.34	7.88	3.08	6.26
Taiwan	10.14	3.32	8.53	2.52	7.85
Korea	8.05	3.23	10.46	2.87	8.87

Table 1: East Asia (cont)

	$\Delta(Y/L)$	$\Delta(I/Y)$		$\Delta(z)$	$\Delta(\hat{z})$
	Data	Data	Model	Model	Model
Country					
Sing.	6.6	1.65	1.77	1.15	2.02
H.K.	9.09	0.89	1.74	1.20	2.62
Mal.	4.49	1.62	1.53	1.13	1.88
Taiwan	10.14	1.68	1.64	1.22	2.63
Korea	8.05	2.67	2.01	1.17	2.08

Table 2: Latin America

	$\Delta(Y/L)$	Years of Schooling				X/Y	
	Data	Data		Model		Model	
Country	1960	1960	2000	1960	2000	1960	2000
Argentina	1.37	4.99	8.49	4.32	6.52	2.1	3.6
Brazil	2.61	2.83	4.56	2.42	3.74	2.3	3.1
Chile	2.14	4.99	7.89	3.87	6.11	2.3	3.5
Colombia	1.39	2.97	5.01	3.11	3.92	2.4	2.9
Costa Rica	1.31	3.86	6.01	3.24	4.86	2.8	3.2
Ecuador	1.78	2.95	6.52	2.69	4.73	1.9	2.6
Mexico	1.84	2.41	6.73	2.43	5.12	2.0	3.5
Paraguay	1.42	3.35	5.74	3.51	4.01	2.0	2.7
Peru	1.00	3.02	7.33	2.94	3.07	2.1	2.1
Uruguay	1.46	5.03	7.25	5.27	7.02	3.0	3.4
Venezuela	0.70	2.53	5.61	2.31	1.43	2.5	2.2

Table 3: Population Growth Rates and $\Delta(I/Y)$

	Growth Rate		$\Delta(I/Y)$	
Country	1960	2000	Model	Data
Argentina	1.71	1.15	1.02	1.03
Brazil	2.91	1.49	1.05	.94
Chile	2.44	1.37	1.04	.90

Source: Population growth rates from GLOBALIS

Adding Heterogeneity

Table 4: Private Education: Selected Countries

	Schooling				x/y			
	Data		Model		Data		Model	
Country	1960	2000	1960	2000	1960	2000	1960	2000
Argentina	4.99	8.49	4.59	6.72	2.5	4.5	2.2	3.7
Brazil	2.83	4.56	2.57	3.49	3.0	5.1	2.5	3.2
Chile	4.99	7.89	3.58	6.47	2.8	4.6	2.4	3.6

Table 4: (cont)

Country	Δz	$\Delta \hat{z}$
Argentina	1.04	1.11
Brazil	1.11	1.76
Chile	1.10	1.67

Relative Educational Expenditures

Table 5: Relative Expenditures per Student

	x_S/x_P		x_T/x_P	
Country	1960	2000	1960	2000
Argentina	1.46	1.46	3.53	3.53
Brazil	4	1.14	11	13.9
Chile	1.93	1.1	4.74	4.0
U.S.		1.32		2.20

Source: UNESCO

Table 6: Public Education

	Schooling				x/y (in %)		Δz	$\Delta \hat{z}$
	Data		Model		Data=Model			
Country	1960	2000	1960	2000	1960	2000		
Argentina	4.99	8.49	5.07	7.83	2.5	4.5	1.05	1.08
Brazil	2.83	4.56	2.89	4.15	3.0	5.0	1.13	1.65
Chile	4.99	7.89	4.34	6.65	2.8	4.6	1.12	1.54

Private and Public Education with Borrowing Constraints

Table 7: Borrowing Constrained Economy

	Schooling				x/y (in %)			
	Data		Model		Data-public		Model-total	
Country	1960	2000	1960	2000	1960	2000	1960	2000
Argentina	4.99	8.49	4.78	8.40	2.3	3.6	2.5	4.5
Brazil	2.83	4.56	2.82	4.61	2.8	4.0	2.9	4.9
Chile	4.99	7.89	4.38	7.66	2.7	4.0	2.8	4.5

Table 7: (cont)

	Δz	$\Delta \hat{z}$
Country		
Argentina	1.03	1.06
Brazil	1.1	1.53
Chile	1.09	1.41

Table 8: Inequality

	Gini		No Schooling (%)	
	Model		Model	
Country	1960	2000	1960	2000
Argentina	0.36	0.39	22%	8%
Brazil	0.42	0.47	48%	18%
Chile	0.39	0.44	27%	13%

Comments and Extensions

Income Distribution

- Distribution of z_h to match schooling levels

- Calculate the life cycle earnings for each individual.

- Compute the Gini coefficient for this artificial economy at different ages.

- The results are in the following table:

Age	Gini-Data	Gini-Model
25	0.21	0.21
35	0.29	0.28
45	0.35	0.35
55	0.39	0.40

- Change returns to scale.

- Alternative γ

	$\gamma = 0.98$		$\gamma = 0.8$	
age	Gini-Data	Gini-Model	Gini-Data	Gini-Model
25	0.21	0.23	0.21	0.20
35	0.29	0.25	0.29	0.35
45	0.35	0.27	0.35	0.41
55	0.39	0.29	0.39	0.49

Immigrant Evidence I

Fact 1 Immigrants earn initially lower income than comparable natives with the same level of schooling. This wage differential has been increasing over time.

Wage differential (Borjas) -16% (1970) and -30%(1990)

Model: -15% (1970) and - 27% (1990) [Matches s and GDP of origin]

Fact 2 The growth rate of earnings of immigrants is higher than the growth rate of earnings of similar –in terms of measurable characteristics— natives.

Frist decade 6-15% higher, second decade 10-25%

Model

Table 7. Growth Rate of Relative Income		
GNP (origin)	10-year	20-year
Middle Income	5-14%	7-20%
Low Income	8-19%	11-27%

Fact 3 The level of earnings of recent immigrants, holding schooling constant, is positively related to the level of per capita output in their country of origin.

Immigrant Evidence II

- In 2000, Mexican GDP per worker was around 40% of that of the US.

- Years of schooling in Mexico were around 6 while that in the US was around 12.

- Chiquiar and Hanson (2005): average wage of a high school graduate who migrated to the United States relative to the average wage of a high school graduate who chose to stay back in Mexico at age 30 is around 1.43.

- Use relative TFP (no adjustment for quality)

$$\frac{w^{US}}{w^{MEX}} = \left(\frac{z^{US}}{z^{MEX}}\right)^{1/(1-\theta)} = (1.136)^{1/(1-0.315)} = 1.2.$$

- Unexpected migration at age 21 (+ adjustment once in the U.S.)

$$\left(\frac{w_{Mig}^{HS}}{w_{\text{Re}\,s}^{HS}}\right)_{Age=30} = 1.34.$$

- Expected migration:

$$\left(\frac{w_{Mig}^{HS}}{w_{\text{Re}\,s}^{HS}}\right)_{Age=30} = 1.93.$$

- Note if immigrant "learns" at age 18 that he will migrate at age 21, then the model predicts exactly 43% wage differential.

Learning by Doing

- Exogenous fertility (for now)

- $$\max \int_I^T e^{-\rho(a-I)} u(c(a), l(a)) da +$$
  $$e^{-\alpha_0+\alpha_1 f} \int_B^{B+I} e^{-\rho(a-I)} u(c_k(a), l_k(a)) da$$
  $$+ e^{-\alpha_0+\alpha_1 f} e^{-\rho B} V^k(h_k(B+I), b_k)$$

subject to

$$
\begin{aligned}
& \int_I^T e^{-r(a-I)} c(a)da + e^f \int_B^{B+I} e^{-r(a-I)} c_k(a)da \\
& + \int_I^R e^{-r(a-I)} x(a)da + e^f \int_B^{B+I} e^{-r(a-I)} x_k(a)da \\
& + e^f e^{-rB} b_k + e^f e^{-r(B+6)} x_E \\
\leq \; & \int_I^R e^{-r(a-I)} wh(a)(1-l(a))da \\
& + e^f \int_{B+S}^{B+I} e^{-r(a-I)} wh_k(a)(1-l_k(a))da + b,
\end{aligned}
$$

subject to

$$
\begin{aligned}
\dot{h}(a) &= z_h[(1 - l(a))h(a)]^{\varsigma_1} x_k(a)^{\varsigma_2} - \delta_h h(a), \\
\dot{h}_k(a) &= z_h \left(n_k(a) h_k(a)\right)^{\gamma_1} x_k(a)^{\gamma_2} - \delta_h h_k(a), \\
& h_k(B+6) = h_I x_E^{\upsilon}, \\
& h(I) \qquad given,
\end{aligned}
$$

- Calibration: $\varsigma = \varsigma_1 + \varsigma_2 = 0.97$.

- Empirical estimates (French and Farber and Gibbons)

Decile	TFP-Baseline	TFP-LBD
90-100	0.99	0.99
40-50	0.90	0.92
0-10	0.73	0.76

Mincer and Ben Porath I

Human Capital (relative to U.S.)			
Decile	$\bar{h}$ (Hall-Jones)	$\bar{h}$ (10%)	$\bar{h}$ (this paper)
90-100	0.93	0.90	0.95
80-90	0.87	0.82	0.88
70-80	0.86	0.80	0.79
60-70	0.80	0.72	0.71
50-60	0.77	0.68	0.60
40-50	0.73	0.64	0.50
30-40	0.62	0.54	0.43
20-30	0.58	0.51	0.32
10-20	0.54	0.48	0.20
0-10	0.42	0.40	0.08

Miracles: Gradual Change in TFP

- Calibrate TFP (in 1960) to match relative (to the U.S.) output in Hong Kong, Korea, Malaysia, Singapore and Taiwan.
- Use actual demographic changes.
- Pick TFP to match smoothed out version of output from 1960 to 2000.
- Can the model match?
  - Schooling
  - Investment-output ratio
- What does it say about TFP?

The Making of the Miracle: Gradual Change in TFP

	Δ(Y/L)	Years of Schooling				Δ(I/Y)		$\Delta(z)$	$\Delta(\hat{z})$
	Data	Data		Model		Data	Model	Model	Model
Country		1960	2000	1960	2000				
Sing.	6.6	3.14	8.12	3.32	8.74	1.65	1.83	1.15	2.08
H.K.	9.09	4.74	9.47	4.42	8.95	0.89	1.91	1.20	2.71
Mal.	4.49	2.34	7.88	3.02	6.32	1.62	1.49	1.13	1.97
Taiwan	10.14	3.32	8.53	2.58	7.93	1.68	1.72	1.22	2.86
Korea	8.05	3.23	10.46	2.99	8.93	2.67	2.13	1.17	2.19

What is the TFP shock?

- Experiments so far assume exogenous TFP movements.
- One possibility - opening up the economy resulted in this large effect.
- We now study the effect of opening up the economy in 1960.
- We also allow demographics to change but TFP remains constant

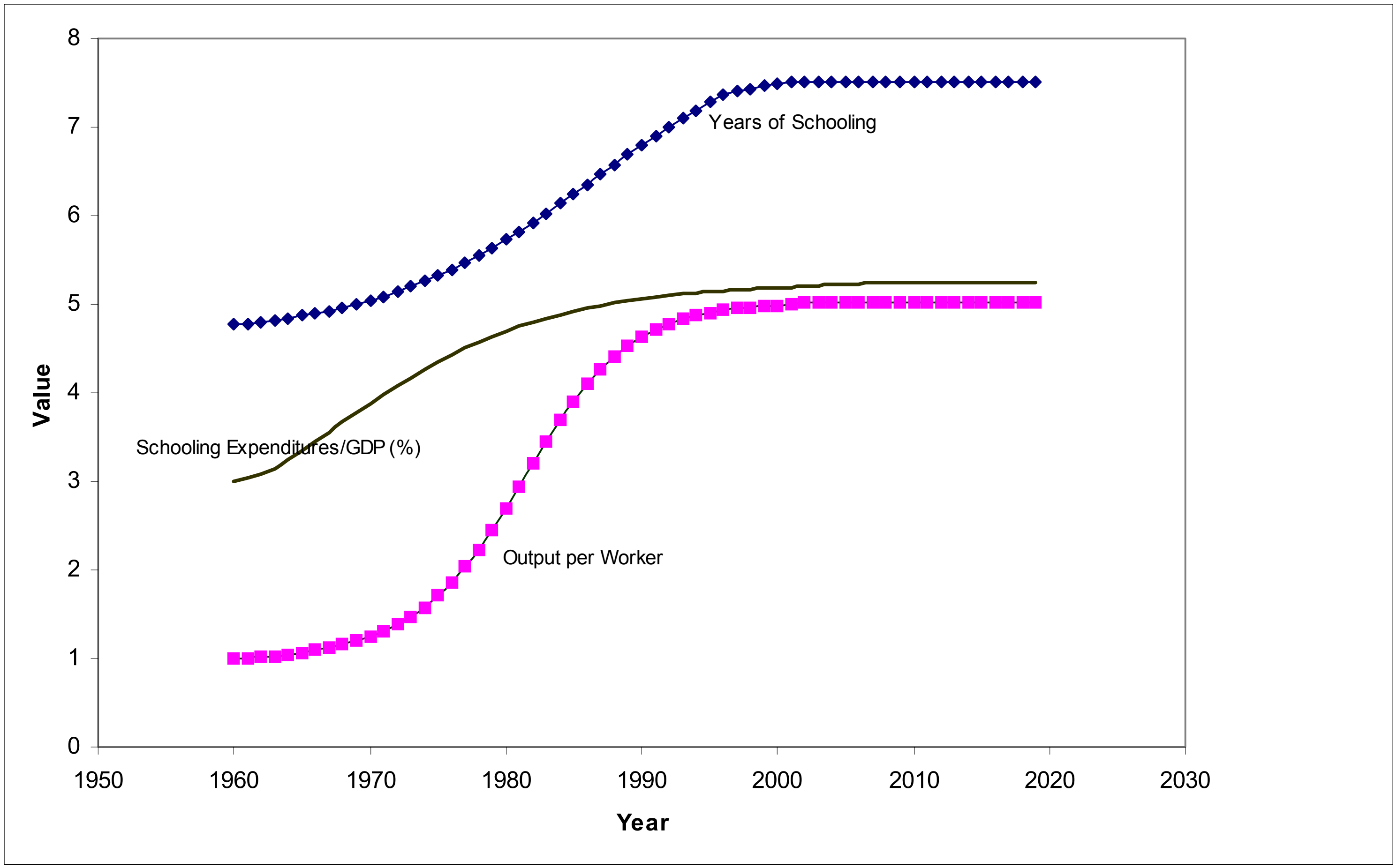
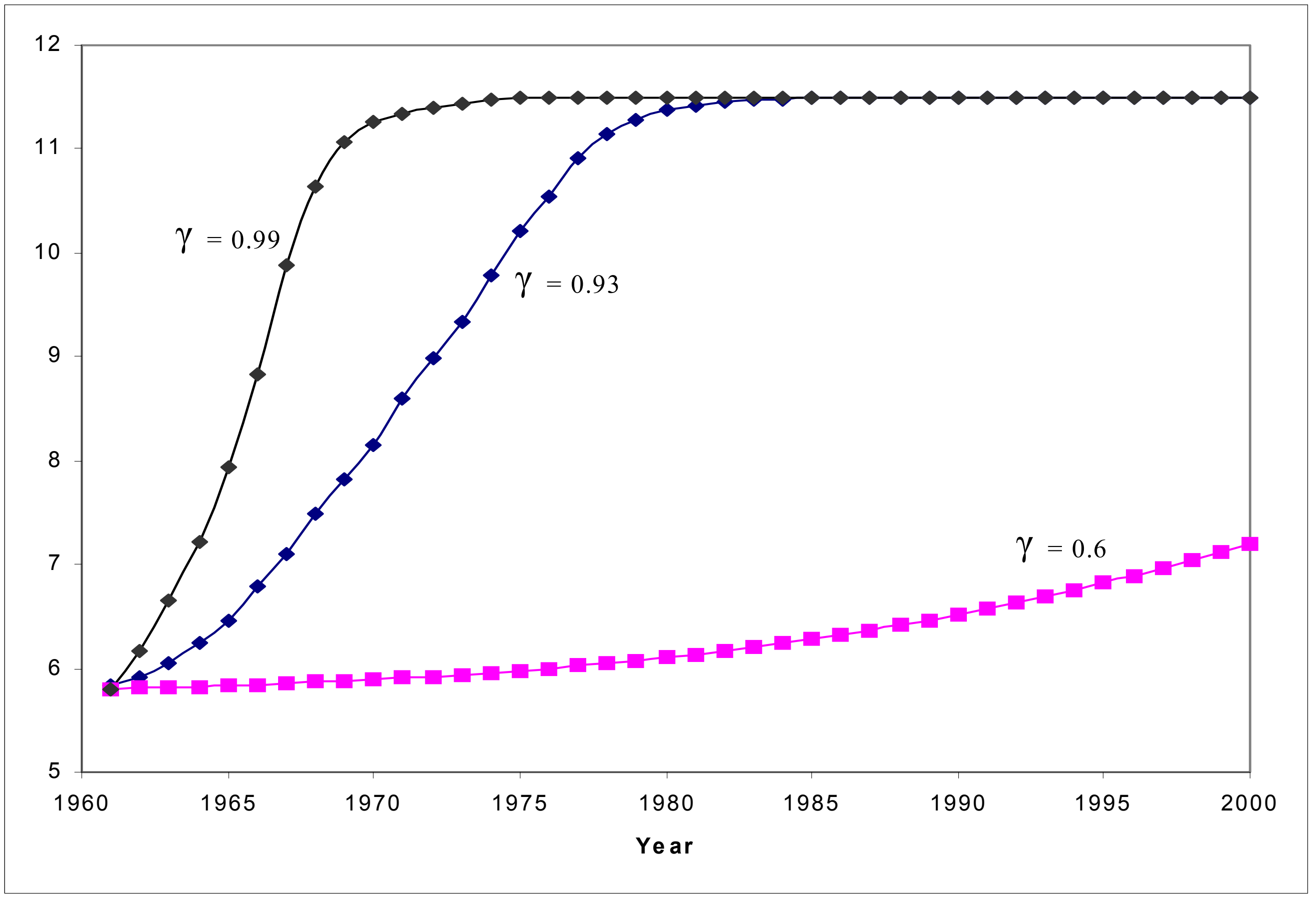


Figure 2: Effects of Opening up the economy



Effect of a one time TFP shock on output per worker - the role played by γ

Infinite Horizon Ben-Porath II

- Model (add market goods)

$$\begin{aligned} y &= zk^{\theta}((1-n)h)^{1-\theta} \\ h' &= (1-\delta_h)h + z_h(nh)^{\gamma_1}x^{\gamma_2} \end{aligned}$$

- Steady state

$$\begin{aligned} n &= \gamma_1 \frac{\delta_h}{\rho+\delta_h} \\ \rho+\delta_k &= \theta z \kappa^{\theta-1} \\ h &= H z^{\frac{\gamma_2}{(1-\theta)(1-\gamma)}} \\ \frac{x/n}{y} &= \frac{\gamma_2}{\gamma_1}\frac{1-\theta}{1-n} \end{aligned}$$

- Schooling (n?) is independent of z

- Quality (h) depends on z.

- Calibration. Assume $\delta_h = 0.03$ (1/40+), $\rho = 0.04$

$$\begin{aligned} n &= .30 = \gamma_1 \times 0.43 \rightarrow \gamma_1 = 0.71 \\ \frac{x/n}{y} &= 0.21 = \frac{\gamma_2(1-\theta)}{\gamma_1(1-n)} \rightarrow \gamma_2 = 0.16 \end{aligned}$$

- Steady state output

$$y = Y_0 z^{\frac{1-\gamma_1}{(1-\theta)(1-\gamma)}} = Y_0 z^{3.49}$$

$$\frac{z_P}{z_{US}} = (0.05)^{1/3.49} = 0.42$$

- Alternative calibration. Assume $\delta_h = 0.03$ $(1/40+)$, $\rho = 0.04$

- $\gamma_1 = 0.78, \ \gamma_2 = 0.18$

$$\frac{z_P}{z_{US}} = (0.05)^{1/8.59} = 0.70$$

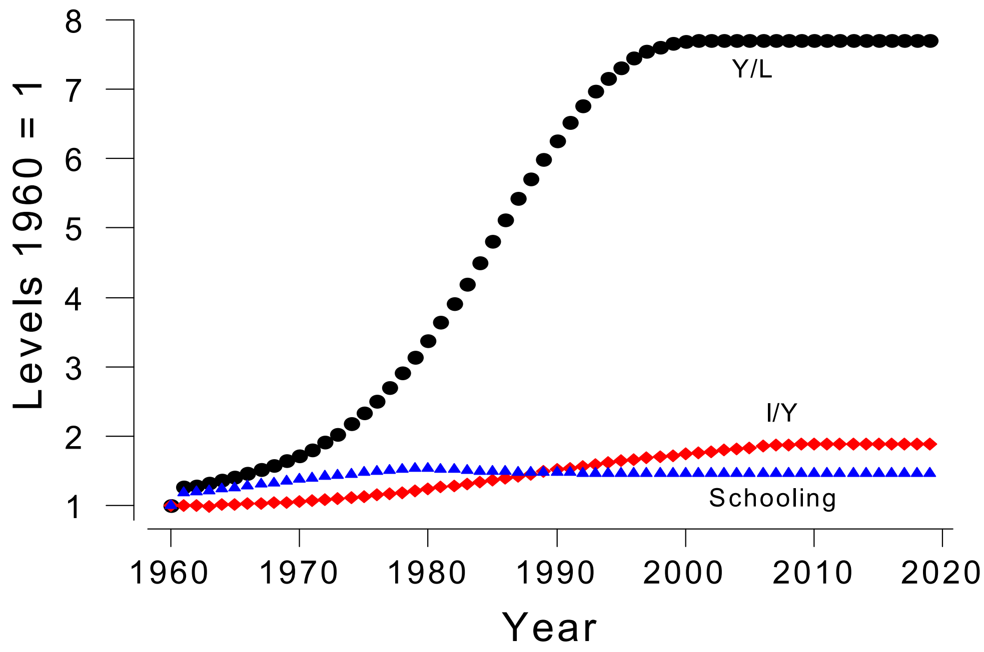
The "Average" Miracle

- Calibrate TFP to the average (in 1960) of Singapore, Hong Kong, Malaysia, Taiwan and Korea.

- Let fertility change: 3.83% in 1960 and 1.68% in 2000.

- Pick a (once and for all) TFP shock such that, in the long run, increase in output per worker matches (7.7)

- How much does each shock contribute?



TFP and Fertility Shocks: Transitional Dynamics

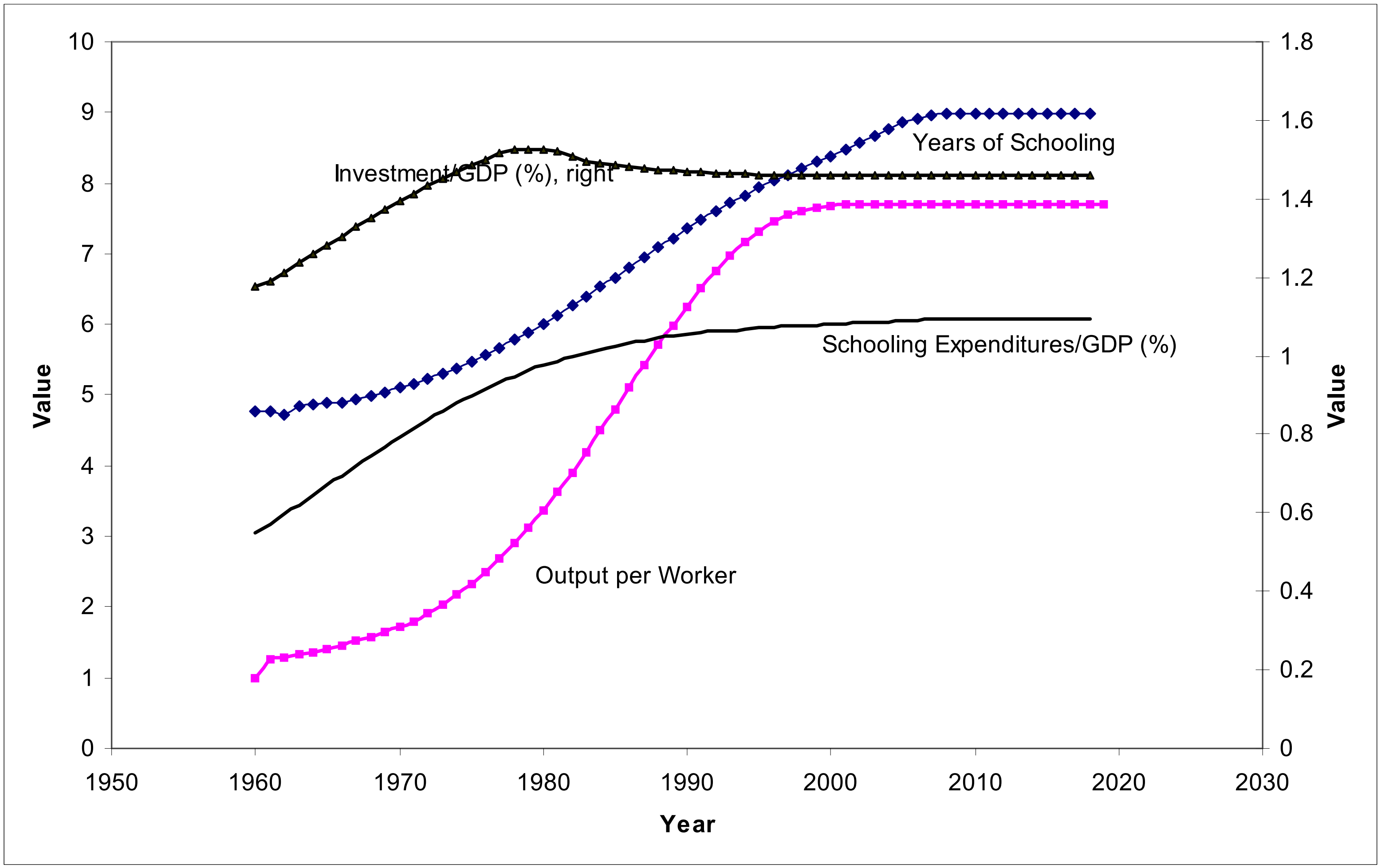
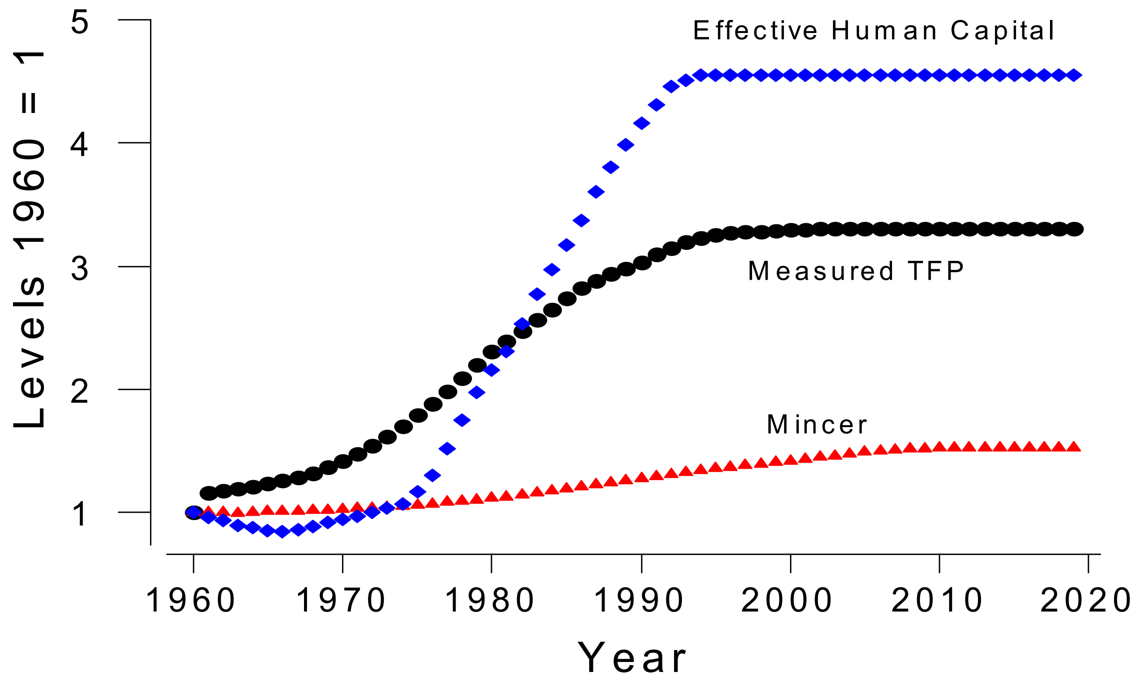
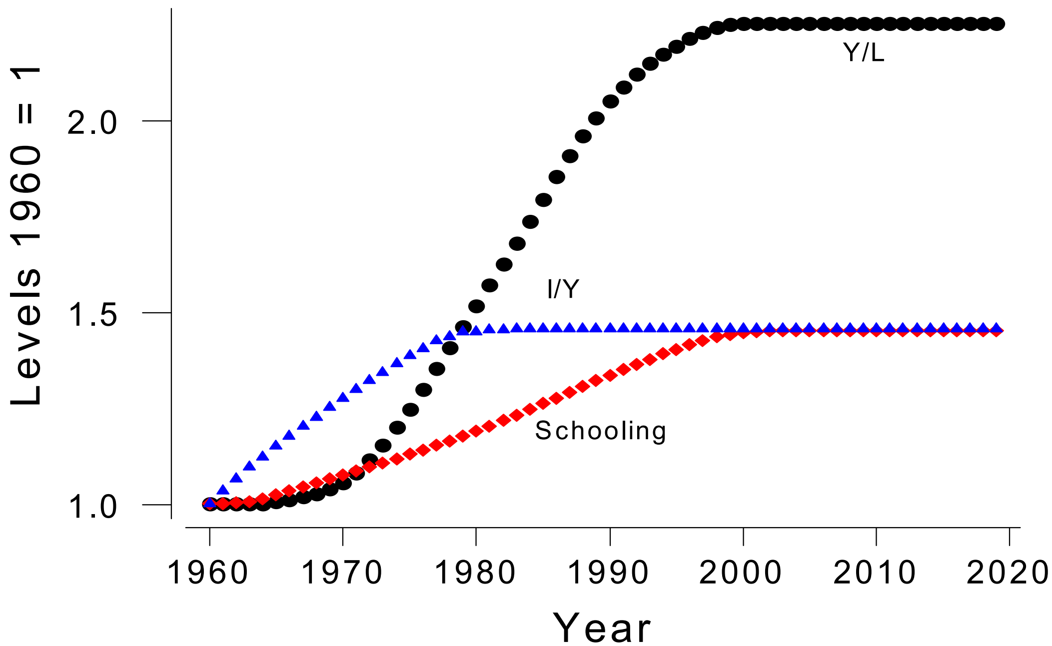


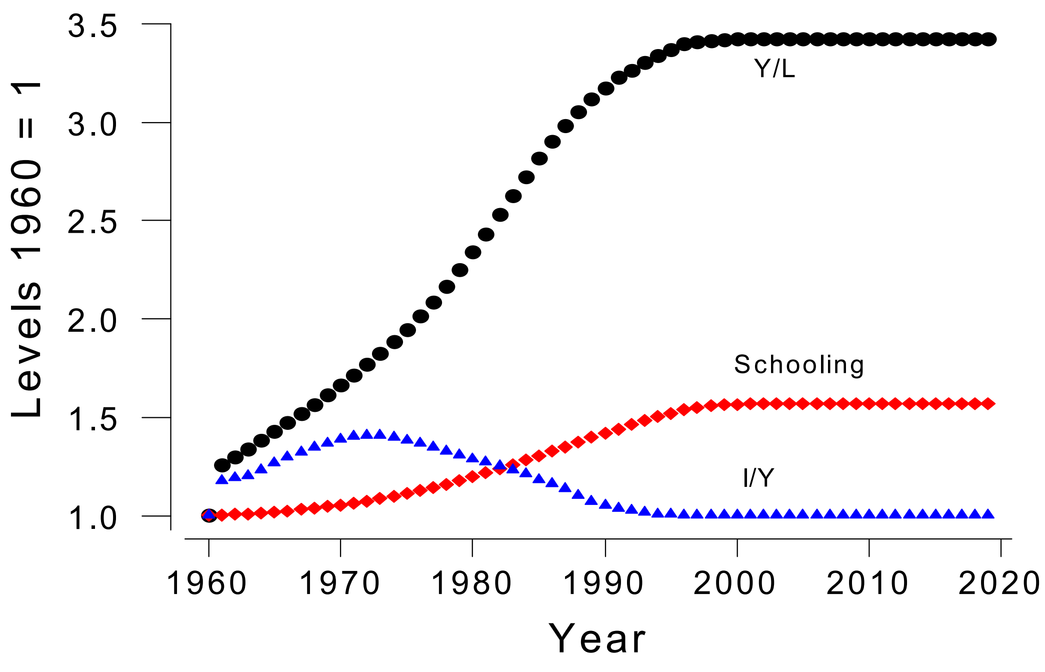
Figure 3: Transitional Dynamics - One time Shock to TFP



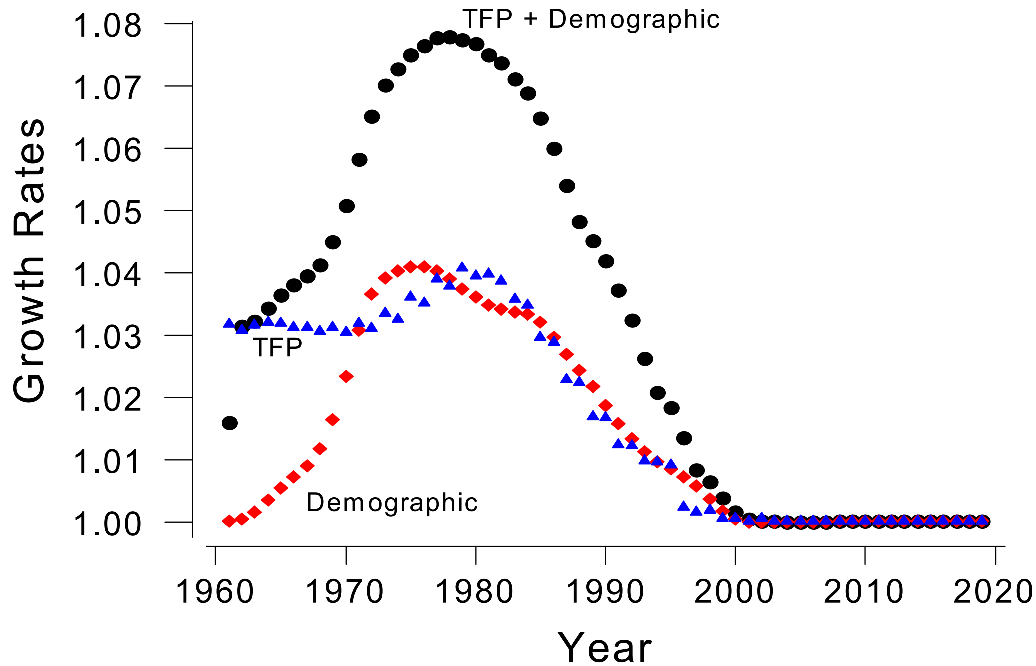
Fast Growers: Measured TFP, Mincerian and Effective Human Capital



Fertility Shock: Transitional Dynamics



TFP Shock: Transitional Dynamics



TFP and Demographic Shocks: Dynamics of the Growth Rate of Output per Worker

Learning by Doing

- Preferences

$$\max \int_I^T e^{-\rho(a-I)} u(c(a), l(a)) da +$$
$$e^{-\alpha_0+\alpha_1 f} \int_B^{B+I} e^{-\rho(a-I)} u(c_k(a), l_k(a)) da$$
$$+ e^{-\alpha_0+\alpha_1 f} e^{-\rho B} V^k(h_k(B+I), b_k)$$

subject to

$$\begin{aligned}
& \int_I^T e^{-r(a-I)} c(a)da + e^f \int_B^{B+I} e^{-r(a-I)} c_k(a)da \\
& + \int_I^R e^{-r(a-I)} x(a)da + e^f \int_B^{B+I} e^{-r(a-I)} x_k(a)da \\
& + e^f e^{-rB} b_k + e^f e^{-r(B+6)} x_E \\
\leq \; & \int_I^R e^{-r(a-I)} wh(a)(1-l(a))da \\
& + e^f \int_{B+S}^{B+I} e^{-r(a-I)} wh_k(a)(1-l_k(a))da + b,
\end{aligned}$$

subject to

$$
\begin{aligned}
\dot{h}(a) &= z_h[(1 - l(a))h(a)]^{\varsigma_1} x_k(a)^{\varsigma_2} - \delta_h h(a), \\
\dot{h}_k(a) &= z_h\,(n_k(a)h_k(a))^{\gamma_1}\,x_k(a)^{\gamma_2} - \delta_h h_k(a), \\
& h_k(B+6) = h_I x_E^{\upsilon}, \\
& h(I) \qquad given,
\end{aligned}
$$

- Calibration: $\varsigma = \varsigma_1 + \varsigma_2 = 0.97$.

- Empirical estimates (French and Farber and Gibbons)

Decile	TFP-Baseline	TFP-LBD
90-100	0.99	0.99
40-50	0.90	0.92
0-10	0.73	0.76

Comparison with other estimates

- Many cite Browning Hansen and Heckman (1999) arguing that the estimates have a wide range
  - Between 0.5 and 1.0
- What explains such a wide range of estimates?
- Generally, much worse data, have only a cross section and not a panel of earnings
  - Heckman (1976) uses a 19% real interest rate ($\gamma = 0.812$)
  - Brown (1976) uses a 33% real interest rate ($\gamma = 0.56$)

- Haley (1976) is simply unable to numerically solve the model for high values of γ ($\gamma = 0.58$)
  - Further, he uses total income as opposed to labor earnings
  - Arbitrarily restricts a key parameter ($\frac{z_h}{h_B^{1-\gamma}}$)

- Browning, Hansen and Heckman (1999) spend the next 8 pages on their preferred model, borrowed from Heckman Lochner and Taber (1998)
  - Heckman Lochner and Taber (1998) estimate γ to be between 0.92 & 0.93!
  - Identification involves both cross-sectional data and age-earnings profiles

- More recent estimates with better data are consistent with a high returns to scale

- Interestingly, Lucas (1988) defended his choice of CRS by appealing to an 'estimate' by Rosen
  - But estimate was for life-cycle model and Lucas's was an infinite-horizon model.
  - Implicit in Lucas (1988) is that the child inherits the human capital of the parent!
- Parente and Prescott (2000) calibrate the returns to scale to be 0.6
  - Use infinite horizon version of Ben-Porath
  - Need lower returns to scale to match data on time devoted to human capital accumulation

The Mincer Earnings Regression - US

- What is the Mincerian return predicted by the model?

- Imagine heterogeneity in z_h, h_B generate heterogeneity in schooling.

- Choose the population weights for the different schooling-experience categories

- Model predicts 8.3% rate of return.

- Bottomline: The Mincer relationship is (approximately) an *equilibrium relationship*!

The Mincer Earnings Regression - International Data

- Compute Mincerian rate of return for each decile
- Regress this return against GDP per capita.
- Model implies negative relationship - Mincer return higher in poorer countries
- Consistent with available evidence.

Distortions and TFP

- Technology

$$y_{it} = Ak_{it}^{\alpha} n_{it}^{\theta} a_i^{1-\alpha-\theta} \qquad 0 < (\alpha, \theta) < 1, \qquad \alpha + \theta < 1,$$

- a_i is interpreted as managerial ability

- $\theta_i = 1$ for all i.

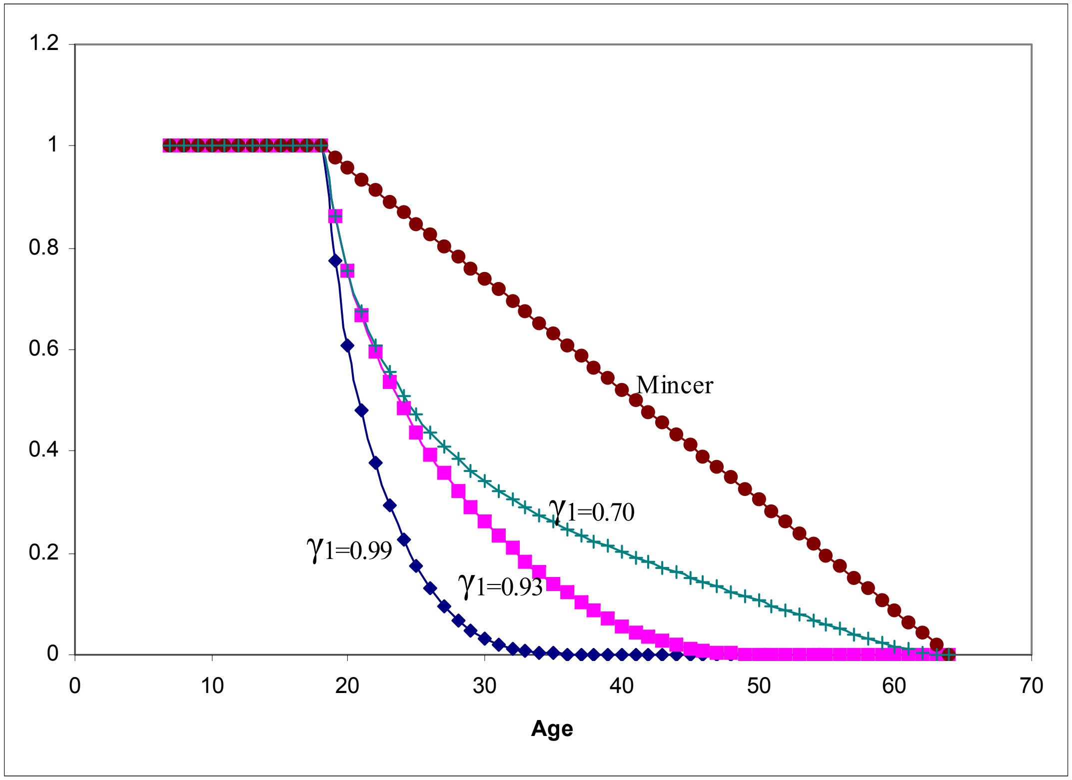


Figure 4: Time Investments for Different Values of γ

Distortionary Policy

- Sector specific taxes and/or subsidies

- If market price is p, a producer in sector i faces a price equal to $p/(1-\tau_i^j)$,

- Solution

$$\begin{aligned} k_{it} &= (1-\tau_{it}^n)^{\frac{\theta}{1-\alpha-\theta}}(1-\tau_{it}^k)^{\frac{1-\theta}{1-\alpha-\theta}}C_k \\ n_{it} &= (1-\tau_{it}^n)^{\frac{1-\alpha}{1-\alpha-\theta}}(1-\tau_{it}^k)^{\frac{\alpha}{1-\alpha-\theta}}C_n \end{aligned}$$

  where C_k and C_n are constants independent of i.

- Equilibrium: economy-wide values denoted by k_t and n_t, respectively.

- View a particular pair $(1-\tau_{it}^{n}, 1-\tau_{it}^{k})$ as being drawn from some joint distribution.

$$\begin{aligned} k_{it} &= k_t(1-\tau_{it}^{n})^{\frac{\theta}{1-\alpha-\theta}}(1-\tau_{it}^{k})^{\frac{1-\theta}{1-\alpha-\theta}}E[(1-\tau_{it}^{n})^{\frac{\theta}{1-\alpha-\theta}}(1-\tau_{it}^{k})^{\frac{1-\theta}{1-\alpha-\theta}}]^{-1} \\ n_{it} &= n_t(1-\tau_{it}^{n})^{\frac{1-\alpha}{1-\alpha-\theta}}(1-\tau_{it}^{k})^{\frac{\alpha}{1-\alpha-\theta}}E[(1-\tau_{it}^{n})^{\frac{1-\alpha}{1-\alpha-\theta}}(1-\tau_{it}^{k})^{\frac{\alpha}{1-\alpha-\theta}}]^{-1} \end{aligned}$$

- Aggregate output

$$y_t = \Delta(\sigma_n, \sigma_k, \rho) A k_t^{\alpha} n_t^{\theta}$$

where

$$\begin{aligned} \Delta(\sigma_n, \sigma_k, \rho) &= \exp\{-\frac{1}{(1-\alpha-\theta)}[\frac{\theta(1-\alpha)\sigma_n^2}{2} \\ &\quad + \frac{\alpha(1-\theta)\sigma_k^2}{2} + \rho\alpha\theta\sigma_n\sigma_k]\} \end{aligned}$$

- If either σ_n or σ_k are positive, $\Delta(\sigma_n, \sigma_k, \rho) < 1$, and *actual* TFP falls short *potential* TFP.

- The μ_j do not enter in $\Delta(\sigma_n, \sigma_k, \rho)$

- $\Delta(\sigma_n, \sigma_k, \rho)$ is strictly convex in $(\sigma_n, \sigma_k, \rho)$.

- Example: ($\alpha = \theta = 0.4$), and $\sigma_n = \sigma_k = \sigma$.

- The values of (σ_n, σ_k) should be interpreted as measures of the cross-sectional variability of incentives relative to the mean level of distortion. Thus, a value of 0.5 corresponds to the case in which the coefficient of variation is 50%. I considered values of σ in the interval [0.1,0.7] with increment size equal to 0.1, and several values of the correlation coefficient ρ. The results are in Table 1

Table 1: TFP Gap						
σ			ρ			
	-0.8	-0,4	0.0	0.4	0.8	1.0
0.1	.99	.99	.98	.98	.98	.98
0.2	.98	.97	.95	.94	.93	.92
0.3	.95	.92	.90	.87	.85	.83
0.4	.91	.87	.83	.78	.74	.73
0.5	.87	.80	.75	.68	.63	.61
0.6	.82	.73	.65	.58	.52	.49
0.7	.76	.65	.56	.47	.41	.38

- Let κ_{it} be the capital-labor ratio in sector i at time t.

$$\kappa_{it} = C \frac{k_t}{n_t} \frac{1 - \tau_{it}^k}{1 - \tau_{it}^n}$$

where C is a constant.

- Let the variance of the log of κ_{it} be denoted $\sigma^2(\ln \kappa_{it})$. Then it follows that,

$$\sigma^2(\ln \kappa_{it}) = \sigma_k^2 + \sigma_n^2 - 2\rho\sigma_k\sigma_n.$$

Thus, if the is no cross-sectional variability in tax/subsidies, $\sigma^2(\ln \kappa_{it})$ should be zero. Evidence of variability, and especially changes over time, is indirect evidence for the presence of distortions.

- Another variable that captures the relevant features of the tax code is the unit price of the sector-specific resource, a_i. The variance of $\ln p_{it}$ is given by,

$$\begin{aligned}\sigma^2(\ln p_{it}) &= \left(\frac{\alpha}{1-\alpha-\theta}\right)^2 \sigma_k^2 + \\ &\left(\frac{\theta}{1-\alpha-\theta}\right)^2 \sigma_n^2 - \frac{2\alpha\theta}{(1-\alpha-\theta)^2}\rho\sigma_k\sigma_n.\end{aligned}$$